Santa Rosa Junior College
Civil & Surveying Technology
CEST 63 Subdivision Planning

Land Surveying Mathematics Handout
Intersection Calculations

Points that identify locations of objects or features, by using northings, eastings, and elevations, is the basis of all land development projects. After the control traverse has been run and balanced, a set of control points with northings, eastings and elevations are used to design and layout the development. Frequently in surveying calculations not all of the data is known to determine the position of points. Other elements must be found to complete the computations. Examples of these calculations are:

- determining the bearing and distance of the "closing leg" of a traverse,
- or determining the distances from two points to a third point,
- or the directions from two points to a third point,
- or a combination of knowing the direction from one point and the distance from another point to a third point.

Each of these cases involves an "intersection" solution requiring a slightly different approach to its solution than what has been presented before.

One approach to solving intersection problems is based on a technique using the coordinates two known points and solving triangles by applying the laws of sines, cosines and right triangle trigonometry to determine the coordinates of the third or intersected point. All of these solutions can be done using a hand held calculator. Most civil engineering and land surveying software packages include COGO routines to make these solutions rapidly. When making hand calculations, it is important to keep track of the algebraic sign of various terms. One way of eliminating this type of blunder is to use the azimuth for all directions instead of the bearing.

Since the computations are based off the known position of points, it is essential that their coordinates be accurately determined. It is always good practice to make an accurate sketch of the problem to help facilitate its solution. The four intersections problems covered are:

1. ***Inverse*** (determine the bearing and distance of the line between two known points)
2. ***Bearing-Bearing Intersection*** (determine the distances of intersection lines when the directions of the lines are known)
3. ***Bearing-Distance Intersection*** (determine the distance and direction of intersecting lines when distance of one line and the direction of another line are known)
4. ***Distance-Distance Intersection*** (determine the direction of intersection lines when the distances from two points are known)

Definitions of intersection notations:

ΔN The difference in northings. Found by subtracting the beginning point northing from the ending point northing.

ΔE The difference in eastings. Found by subtracting the beginning point easting from the ending point easting.

Az_1 The azimuth of the first course. Direction is from the first known point to intersection point.

Az_2 The azimuth of the second course. Direction is from intersection point to the second known point.

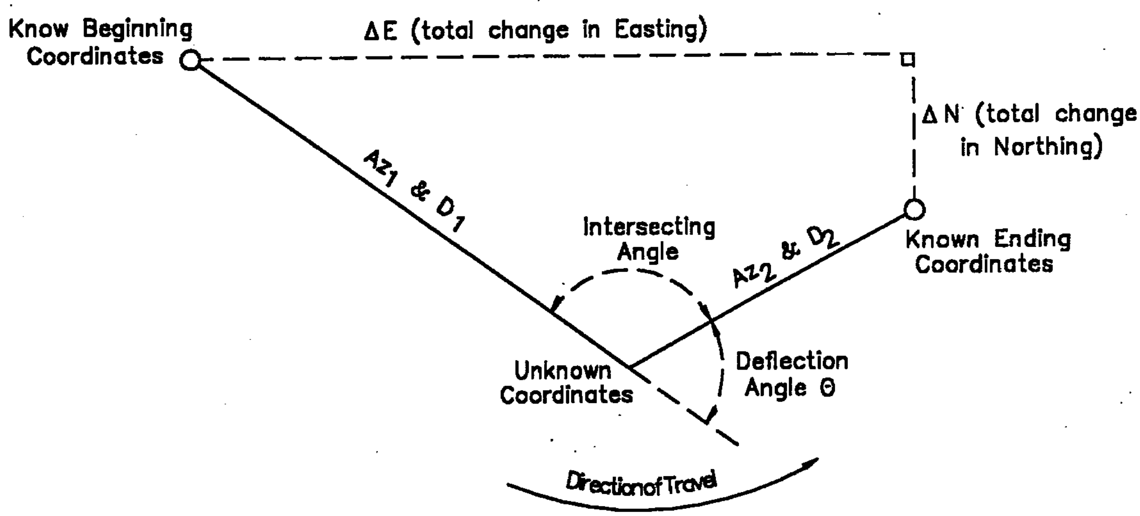
D_1 The distance of the first course.

D_2 The distance of the second course.

θ The deflection angle between the two intersecting lines.

$| \; |$ Any notation bracketed by this symbol stands for the absolute value (or positive value) of that notation.

Diagram of intersection notations:



Note!

The precision desired is generally to the hundredth of a foot or centimeter depending on units. It is customary to let your calculator run out the decimal places as far as possible and round off your answer to two decimal places at the end.

INVERSE **(determine the bearing and distance between two points)**

Given the coordinates of points A and B determine the bearing and distance of the line AB.

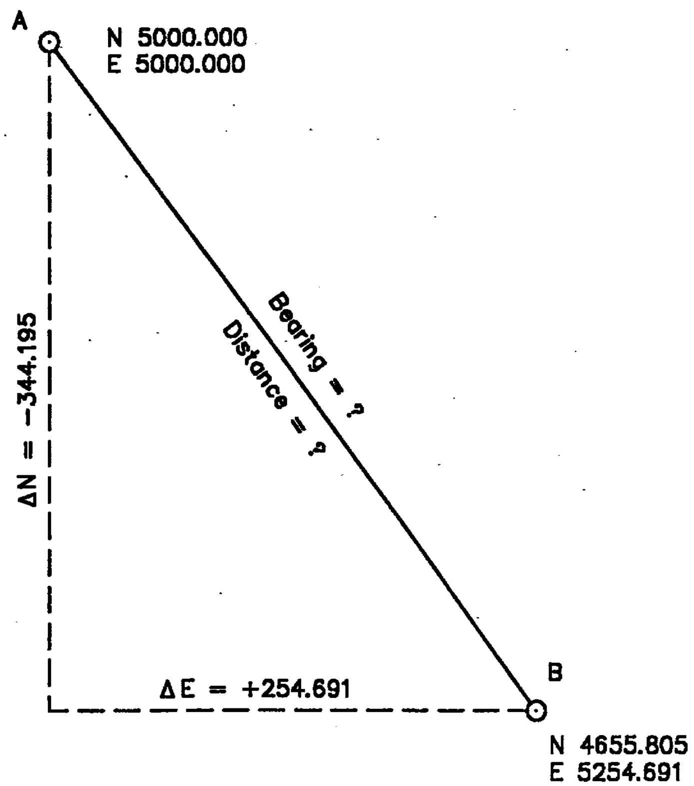
Equations:

$$\text{Bearing Angle}_{AB} = \tan^{-1} \frac{|\Delta E|}{|\Delta N|}$$

Letter designations for the bearing are determined by the magnitude of the northings and eastings from A to B.

$$\text{Distance}_{AB} = \sqrt{(\Delta E)^2 + (\Delta N)^2}$$

Solution:



$\Delta E = +254.691$

$\Delta N = -344.195$

$\text{Bearing Angle}_{AB} = \tan^{-1} (|+254.691| \div |-344.195|) = 36° 30' 00"$

Bearing_{AB} = S 36° 30' 00" E

$\text{Distance}_{AB} = \sqrt{(+254.691)^2 + (-344.195)^2} = 428.180'$

BEARING-BEARING INTERSECTION (determine the distance of the two intersecting lines given the directions from the two known points)

Given the coordinates of points A and B determine the distance AC and the distance CB.

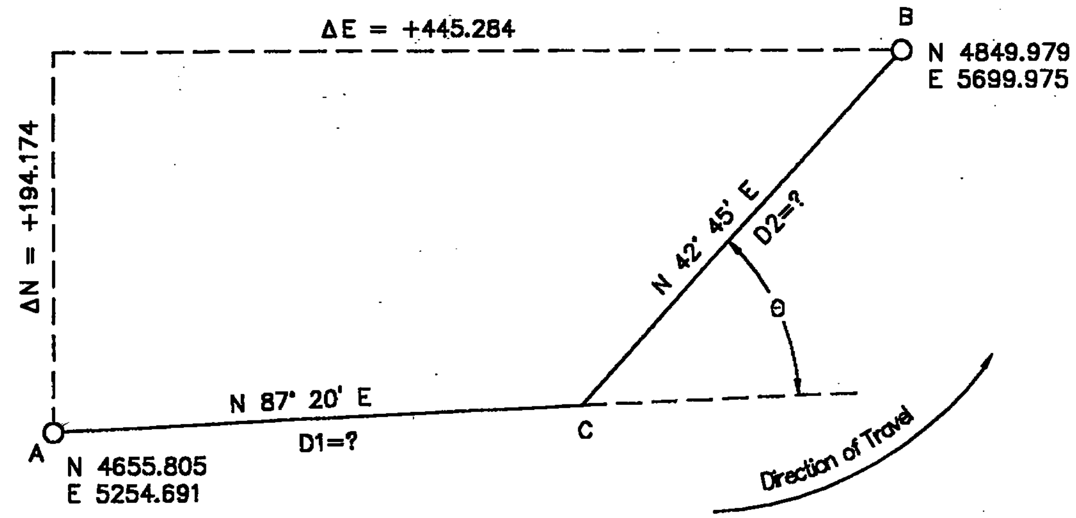
Equations:

$$|D_1| = \frac{(\Delta E \cos Az_2) - (\Delta N \sin Az_2)}{(\sin \theta)}$$

$$|D_2| = \frac{(\Delta E \cos Az_1) - (\Delta N \sin Az_1)}{(\sin \theta)}$$

Solution:

$\Delta E = +445.284$ $\Delta N = +194.174$ $\theta = 44° 35' 00''$ $Az_1 = 87° 20' 00''$ $Az_2 = 42° 45' 00''$



$$|D_1| = \frac{(+445.284 \cos 42° 45' 00'') - (+194.174 \sin 42° 45' 00'')}{(\sin 44° 35' 00'')} = 278.051'$$

$$|D_2| = \frac{(+445.284 \cos 87° 20' 00'') - (+194.174 \sin 87° 20' 00'')}{(\sin 44° 35' 00'')} = 246.810'$$

Once bearing and distance from AC or BC is determined, then the northings and eastings of point C can be calculated.

BEARING-DISTANCE INTERSECTION (determine the unknown distance and unknown bearing of the intersecting lines given a direction or distance from the two known points)

Given the coordinates of points A and B determine the distance AC and the direction CB.

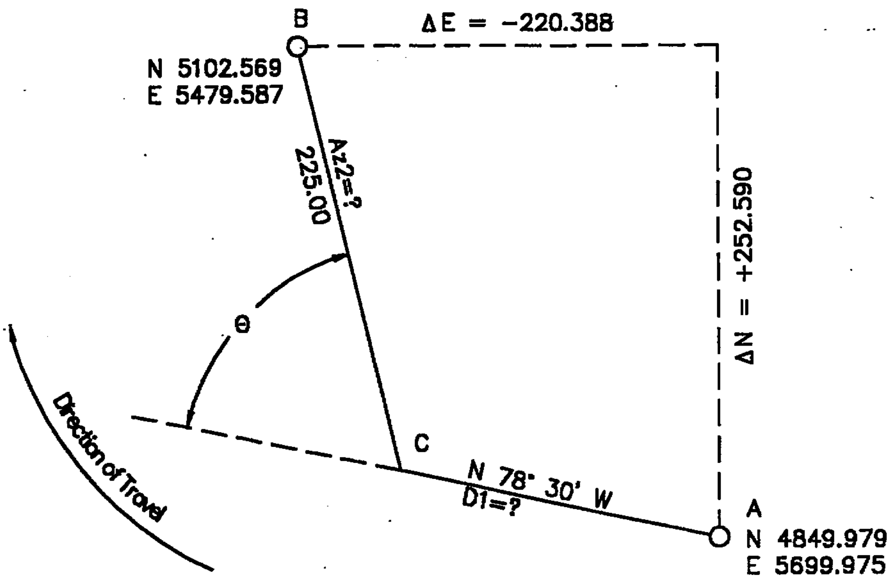
Equations:

$$\theta = \sin^{-1} \frac{(\Delta E \cos Az_1) - (\Delta N \sin Az_1)}{(D_2)}$$

$$|D_1| = \frac{(\Delta E \cos Az_2) - (\Delta N \sin Az_2)}{(\sin \theta)}$$

Solution:

$\Delta E = -220.388$ $\Delta N = +252.590$ $Az_1 = 281° 30' 00''$ $D_2 = 225.00'$



$$\theta = \sin^{-1} \frac{(-220.388 \cos 281° 30' 00'') - (+252.590 \sin 281° 30' 00'')}{(225.00')} = 64° 47' 49''$$

$Az_2 = 281° 30' 00'' + 64° 47' 49'' = 346° 17' 49''$

$$|D_1| = \frac{(-220.388 \cos 346° 17' 49'') - (+252.590 \sin 346° 17' 49'')}{(\sin 64° 47' 49'')} = 170.511'$$

Once bearing and distance from AC or BC is determined, then the northings and eastings of point C can be calculated.

DISTANCE-DISTANCE INTERSECTION **(determine the directions of the intersecting lines given the distances from the two known points)**

Given the coordinates of points A and B determine the direction AC and the direction CB.

Equations:

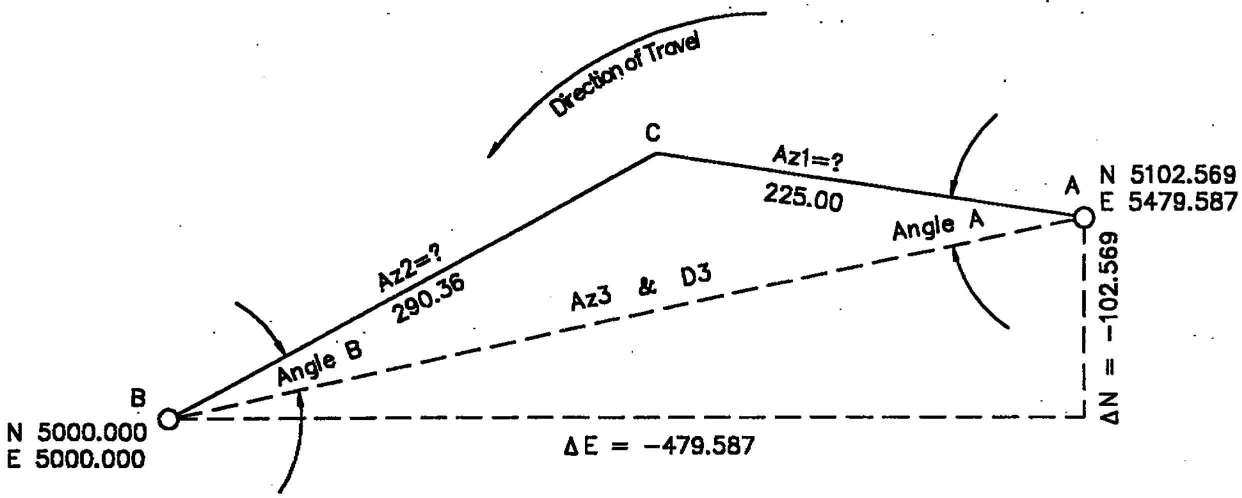
$$S = (D_1 + D_2 + D_3) \div 2$$

$$\text{Angle A} = 2\left(\cos^{-1}\sqrt{\frac{S\,(S - D_2)}{(D_3)\,(D_1)}}\right)$$

$$\text{Angle B} = 2\left(\cos^{-1}\sqrt{\frac{S\,(S - D_1)}{(D_3)\,(D_2)}}\right)$$

Solution:

$\Delta E = -479.587$ $\Delta N = -102.569$ $D_1 = 225.00'$ $D_2 = 290.360'$ $D_3 = 490.433'$



$S = (225.00' + 290.360' + 490.433') \div 2 = 502.896'$

$$\text{Angle A} = 2\cos^{-1}\sqrt{\frac{502.896'\,(502.896' - 290.360')}{(490.433')\,(225.00')}} = 20^\circ\ 24'\ 38''$$

$$\text{Angle B} = 2\cos^{-1}\sqrt{\frac{502.896'\,(502.896' - 225.00')}{(490.433')\,(290.360')}} = 15^\circ\ 40'\ 44''$$

You'll need to compute the bearing of line AB first. Then compute the bearings of AC and BC by using the Angles A and B from above. Once all the bearings and distances are known, then the northings and eastings of point C can be calculated.