Astronomy 253 — Solutions to Group Problems 2

Friday, January 30

1. **(a)** Multiply both sides by $(1 - v/c)$:

$$(1 - v/c)(1 + z) = \sqrt{(1 + v/c)(1 - v/c)}$$

Recognize the right side as the difference of two squares:

$$(1 - v/c)(1 + z) = \sqrt{(1 - (v/c)^2)}$$

Multiply out the left side and use the binomial theorem on the right side:

$$1 - \frac{v}{c} + z - z\frac{v}{c} = 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2$$

Recognize that any term in v/c is $\ll 1$, as is z, so any terim in $(v/c)^2$ or $z(v/c)$ is much smaller, and can be ignored. (If you ignore the terms of order v/c, you get 1=1, which isn't terribly interesting.)

$$1 - \frac{v}{c} + z = 1$$

$$\boxed{z = \frac{v}{c}}$$

(b) The line is shifted to a higher wavelength, which is a shift to the red, or a positive redshift. This indicates that the object is moving *away* from us. The redshift is:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{(6568\text{Å} - 6563\text{Å})}{6563\text{Å}} = \frac{5\text{Å}}{6563\text{Å}} = 7.6 \times 10^{-4}$$

We have $z \ll 1$, so:

$$z = \frac{v}{c}$$

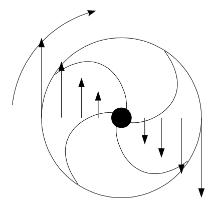
$$v = (7.6 \times 10^{-4})\,(3 \times 10^5\,\text{km/s})$$

$$\boxed{v = 230\,\text{km/s}}$$

(c) No! If we're orbiting in a circle, we're always the same distance from the center of the Galaxy. Thus, no Doppler shift. (And if you want to talk gravitational redshift due to the Galaxy's potential well, you're thinking way too hard.)

(d) Reshifted. If Mars has just passed the closest point to the Sun, then it is moving away from the Sun. (Mostly, it's moving tangent to the Mars-Sun separation, but some small part of its motion is away from the Sun.) Or, to an observer on Mars, the Sun is moving away. Thus, the Sun should show a very small redshift.

2. This one is a little bit tricky, because you have to worry about how much dust absorbs starlight and how far you can see into the galaxy, how the velocities you see along one line of sight vary, etc. But we're not doing complicated modelling here. To make it simple, let's pretend that we can see all they way through the galaxy, so what you see in the middle will be the average velocity. What you see along the middle of any line of site are velocities like the following (with longer arrows indicating higher speed):



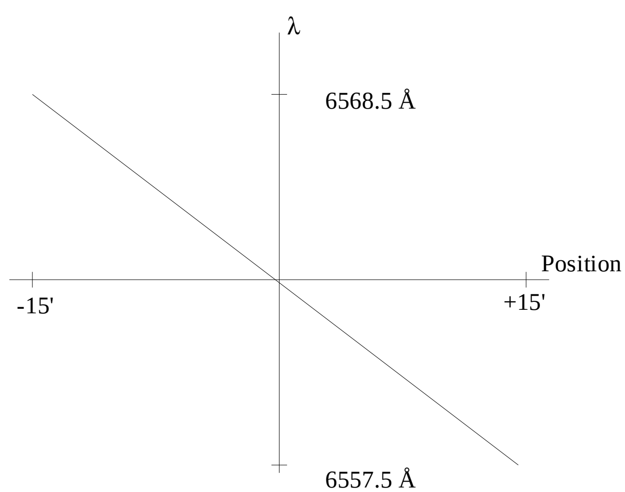
Thus, on the left you will see redshifts (things are moving away), and on the right, blueshifts. If the galaxy is rotating like a rigid object, the points on the outside have to be moving faster (since they have a larger circumference to travel in one rotation). To get the scale of our plot, let's figure out the maximum $\Delta\lambda$:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

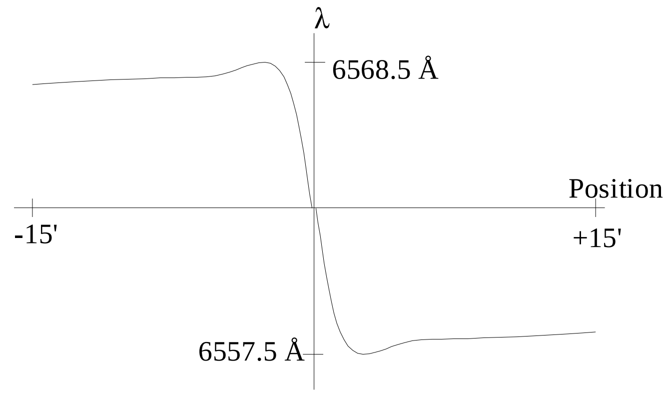
$$\frac{\Delta\lambda}{6563\text{Å}} = \frac{250\,\text{km/s}}{3 \times 10^5\,\text{km/s}}$$

$$\Delta\lambda = (6563\text{Å})\,(8.33 \times 10^{-4}) = 5.5\text{Å}$$

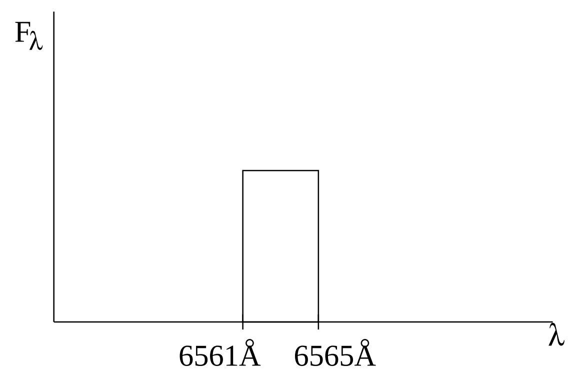
Therefore, we will observe wavelengths between 6568.5 and 6557.5 Å for the Hα line, and our plot will look like:



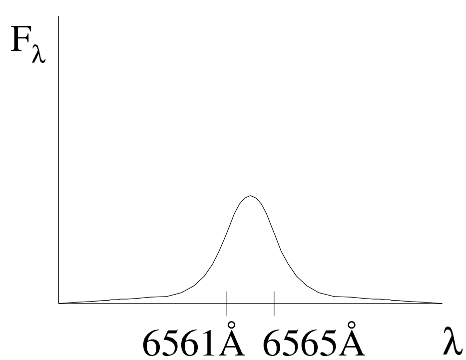
3. Under the same "transparent galaxy" assumption as last time, this problem is basically already done for us! Because $\Delta\lambda$ is proportional to v, the only differences between a plot of velocity and a plot of λ are the scale of the plot and the central point. (Velocity 0, the x-axis on the velocity plot, corresponds to an unshifted wavelength of 6563Å, *not* zero wavelength.) Again, with a redshift on the left and a blueshift on the right, the wavelength vs. position plot will look something like:



4. **(a)** If you have an evenly distributed set of velocities between ± 100 km/s, you will get a line which shifts by up to $\Delta\lambda = \pm(6563\text{Å})(100/3 \times 10^5) = 2\text{Å}$. If all velocities are equally likely, then the line strength in the range will be constant:



(b) A Gaussian distribution of velocities will give you a Gaussian distribution of wavelengths:



(c) As a gas gets hotter, it means that the particles are moving about with faster random velocities. This will tend to lead to a *wider* line, as higher velocities in random directions, some towards you, some away from you, will lead to a larger range of observed Doppler shifts. As for the strength of the line, that's much more complicated.... You might think that temperature would make the overall energy in the line stronger, if (for instance) the upper state of the line is colisionally excited. Higher temperature, faster moving particles, more collisions with more energy. But there could potentially be a lot more going on. Take Astronomy 311 if you want to delve into this subject in detail....

5. **(a)** Gravitational potential energy. That's a lot of mass in that core which is getting ten times bigger. As you move gravitating objects *away* from each other, you are *increasing* their gravitational potential energy. (Think about lifting a rocket of from Earth; to move it away from the Earth takes a lot of energy. If you drop an asteroid on the Earth, things are coming closer together, and a lot of energy is *released*.)

(b) We could spend a lot of time looking up the size of the atomoic mass unit and so forth, but let's stick to basics. Three He-4's combine into one C-12. Notice that three He-4's have *more* mass than one C-12. The extra mass is converted into energy at the rate of $E = mc^2$. The *fraction* of mass converted to energy is:

$$\frac{3\,(4.0026)\; - \;12}{12} \;=\; 6.5\times10^{-4}$$

The total amount of mass converted to energy then is this fraction of the mass that participates in the fusion. Since we're assuming that the core has 1/3 the mass of the Sun, and one tenth of the core participates in the fusion, the total amount of mass converted is:

$$(6.5\times10^{-}4)\,(0.1)\,\left(\frac{2\times10^{30}\,\mathrm{km}}{3}\right) \;=\; 4.3\times10^{25}\,\mathrm{kg}$$

The energy released in fusion is therefore:

$$E = mc^2 \;=\; (4.3\times10^{25}\,\mathrm{kg})(3\times10^{8}\,\mathrm{m/s^2}) \;=\; 4\times10^{42}\,\mathrm{J}$$

On the other side, to *really* do the potential energy right, we'd have to know something about the density structure of the core bofore and afterwards. But, to order of magnitude, the potential energy in a sphere is going to be $-GM^2/R$, where R is the radius of the sphere and M is the mass of the sphere. Our mass is 1/3 the mass of the Sun, and R goes from 7,000 km to 70,000 km, so the potential put into the system is:

$$\frac{-(6.67\times10^{-11}\mathrm{m^3\,kg^{-1}, s^{-2}})\,(2/3\times10^{30}\,\mathrm{kg})^2}{7\times10^{7}\,\mathrm{m}}$$

$$- \quad \frac{-(6.67\times10^{-11}\mathrm{m^3\,kg^{-1}\,s^{-2}})\,(2/3\times10^{30}\,\mathrm{kg})^2}{7\times10^{6}\,\mathrm{m}}$$

$$= 4 \times 10^{42}\,\mathrm{J}$$

So, to order of magnitude, it works out. (It looks better than order of magnitude, but our potential energy calculation really is only good to order of magnitude, so don't overinterpret the numerical conincidence that results from Prof. Knop having chosen the 1/3 and 0.1 numbers to work out about right.)