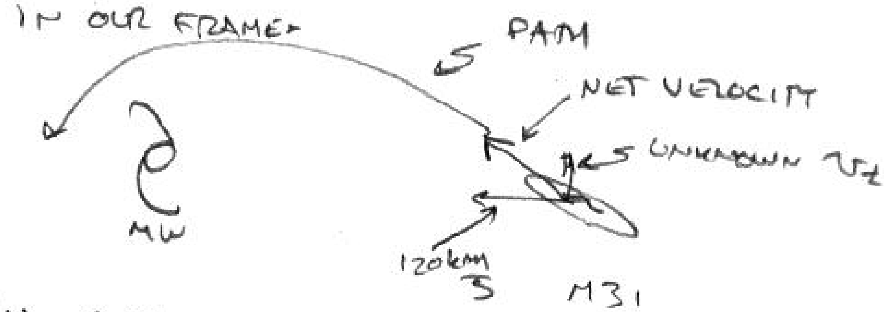
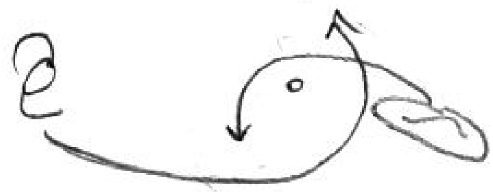
$\boxed{1}$ (a) BECAUSE TANGENTIAL VELOCITIES ARE HARD TO MEASURE



IN C-M. FRAME



(b)

$$\left(\frac{750\,\text{kpc}}{120\,\text{km/s}}\right)\left(\frac{3.1\times10^{16}\,\text{m}}{\text{pc}}\right)\left(\frac{1000\,\text{pc}}{\text{kpc}}\right)\left(\frac{1\,\text{km}}{1000\,\text{m}}\right)$$

$$= 2\times10^{17}\,\text{s} = \boxed{6 \text{ BILLION YEARS}}$$

(c) INEVITABLE

DYNAMIC FRICTION → EACH PASS, BULK KE IS CONVERTED INTO STIRRING UP & FLUFFING UP THE STARS WITHIN EACH GALAXY, AND THE ORBITS DECAY.

2

- GRAVITATIONAL LENSING

  BACKGROUND GALAXIES BENT MORE, MORE MASS IN CLUSTER

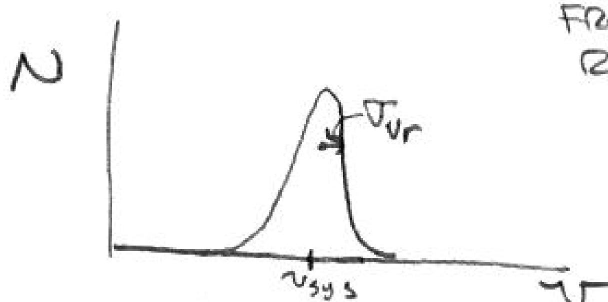
- VIRIAL THEOREM

  MEASURE VELOCITY DISPERSION & SIZE OF CLUSTER

$$\frac{\sqrt{3}}{2} M \sigma_{vr}^2 \sim \underbrace{\frac{G M^2}{R}}$$

$\sqrt{3}$ SINCE WE ONLY

OR USE THE INTEGRATED EXPRESSION FROM A MORE REALISTIC CLUSTER Φ



LINE-OF-SIGHT VELOCITY OF GALAXY IN CLUSTER

3

(a) $M = \frac{4}{3}\pi a^3 \rho_0$

(b) SPHERICAL SYMMETRY, ALL MASS @ SMALLER RADII $\rightarrow$ $F = -\frac{GMm}{r^2}\hat{r} = -m\vec{\nabla}\Phi$ JUST LIKE A POINT SOURCE

SO $\boxed{\Phi = -\frac{GM}{r} = -\frac{4G\pi a^3 \rho_0}{3r}}$

(c) FOR $R<a$, $F = -\frac{GM(<R)m}{r^2}\hat{r} = -\frac{G\frac{4}{3}\pi r^3 \rho_0 m}{r^2}\hat{r}$

$= -\frac{4\pi}{3} G m \rho_0 r \hat{r}$

$\Rightarrow$

TRY $\Phi = \frac{2\pi G \rho_0}{3} r^2 + \text{const}$

$$-m \vec{\nabla} \Phi = -m \frac{\partial}{\partial r}\left(\frac{2\pi G\rho_0}{3} r^2 + \text{const}\right) \hat{r}$$

$$= -m \frac{4}{3}\pi G \rho_0 r \, \hat{r}$$

IT WORKS!

FOR CONTINUITY @ $r = a$,

$$\frac{2\pi}{3} G \rho_0 a^2 + \text{const} = -\frac{4G}{3}\pi a^2 \rho_0$$

$$\boxed{\text{const} = -\frac{2\pi}{3} G \rho_0 a^2}$$

(d)

$$DE = \frac{1}{2}\int \rho \Phi \, d^3r = \frac{1}{2}\int_0^a (\rho_0)\left(\frac{2\pi G\rho_0}{3} r^2 - \frac{2\pi}{3} G\rho_0 a^2\right) 4\pi r^2 dr$$

$$= \frac{1}{2}\int_0^a \frac{8\pi^2 G \rho_0^2}{3} r^4 dr - \frac{1}{2}\int_0^a \frac{8\pi^2 G\rho_0^2 a^2 r^2}{3} dr$$

$$= \frac{4}{15}\pi^2 G \rho_0^2 a^5 - \frac{4}{9}\pi^2 G \rho_0^2 a^5$$

$$= \left(\frac{12 - 20}{45}\right) \pi^2 G \rho_0^2 a^5$$

$$= \boxed{\frac{-8}{45}\pi^2 G \rho_0 a^5}$$

UNLESS I DID SOMETHING STUPID.

(e) FOR $r > a$, $\frac{v_c^2}{r} = \left|\frac{-GM}{r^2}\right|$

$$\boxed{v_c = \sqrt{\frac{GM}{r}} = \sqrt{\frac{4\pi G a^3 \rho_0}{3r}} \quad r \geq a}$$

$\Longrightarrow$

3 (e) cont'd

FOR $r < a$, $\frac{v_c^2}{r} = |\vec{\nabla}\Phi|$

$$= \frac{4\pi}{3} G \rho_0 r$$

$$\boxed{v_c = \sqrt{\frac{4\pi}{3} G \rho_0}\, r \qquad r \leq a}$$

NOTE: @ $r = a$, CHECK THAT BOTH EXPRESSIONS ARE THE SAME! (THEY ARE)

(f) TOTAL KE

$$= \int \underbrace{(\rho d^3 r)}_{dm} v_c^2$$

$$= \int_0^a \rho_0 \left(\frac{4\pi}{3} G \rho_0 r^2\right) 4\pi r^2 dr$$

$$= \frac{16\pi^2 G \rho_0^2}{3} \left(\frac{a^5}{5}\right)$$

$$= \frac{16\pi^2 G \rho_0^2 a^5}{15}$$

$KE > \frac{|PE|}{2}$ SO THIS GALAXY IS <u>NOT</u> IN VIRIAL EQUILIBRIUM!

(UNLESS I DID SOMETHING STUPID)

4

(a)

$$mc^2 = (100\ \mathrm{GeV})\left(\frac{10^9\ \mathrm{eV}}{\mathrm{GeV}}\right)\left(\frac{1\ \mathrm{J}}{6.24\times10^{18}\ \mathrm{eV}}\right) = 1.6\times10^{-8}\ \mathrm{J}$$

$$\boxed{m = 1.78\times10^{-25}\ \mathrm{kg}}$$ ONE D.M. PARTICLE

(b)

WE HAVE

$$\rho = \underset{\substack{\uparrow \\ \text{MASS OF ONE}}}{m}\ \underset{\substack{\uparrow \\ \text{NUMBER DENSITY}}}{n} = \left(\frac{1\ M_\odot}{10\ \mathrm{pc}^3}\right)\left(\frac{2\times10^{30}\ \mathrm{kg}}{M_\odot}\right)\left(\frac{1\ \mathrm{pc}}{3.09\times10^{16}\ \mathrm{m}}\right)^3$$

$$= 6.8\times10^{-21}\ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

THUS

$$\boxed{\begin{aligned} n &= 3.8\times10^{4}\ \mathrm{m}^{-3} \\ &= 0.038\ \mathrm{cm}^{-3}\end{aligned}}$$

(c) VERY ROUGHLY

$$PE \sim \frac{GM^2}{R}$$

USE $R = 50\ \mathrm{kpc}$ INSTEAD OF $100\ \mathrm{kpc}$ SINCE IT TAPERS → WHAT THE HECK

$$G = 6.67\times10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$$

$$PE \sim \left(\frac{-G(10^{12} M_\odot)^2}{50\times10^3\ \mathrm{pc}}\right)\left(\frac{2\times10^{30}\ \mathrm{kg}}{M_\odot}\right)^2\left(\frac{1\ \mathrm{pc}}{3.09\times10^{16}\ \mathrm{m}}\right)$$

$$\boxed{PE \sim -1.7\times10^{53}\ \mathrm{J}}$$

(d)

$$KE = \frac{|PE|}{2} \sim 8\times10^{52}\ \mathrm{J} \sim \frac{1}{2}Mv^2$$

$$v = \sqrt{\frac{1.7\times10^{53}\ \mathrm{J}}{10^{12}\ M_\odot}\left(\frac{1\ M_\odot}{2\times10^{30}\ \mathrm{kg}}\right)}$$

$$\boxed{v = 3\times10^{5}\ \frac{\mathrm{m}}{\mathrm{s}}}$$

NOTE: $v \ll c$ → THIS IS WHY IT'S "COLD" DARK MATTER

4 cont'd

e LETS TRY

$$\frac{1}{2} m v^2 \sim 10^{-12} \underbrace{\frac{e^2}{4\pi\epsilon_0 r}}_{\text{ELECTROMAG PE FOR TWO CHARGES}}$$

(WE COULD ALSO TRY

$$10^{25} \frac{G m^2}{r}$$

AND GET 3×10^{-21} m)

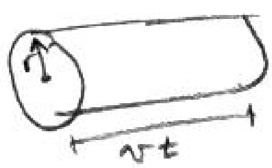
SO THIS ESTIMATE IS GOOD TO WITHIN TEN ORDERS OF MAGNITUDE.... MEH.

$$r \sim \frac{2\times10^{-12} e^2}{4\pi\epsilon_0 m v^2} = \frac{(2\times10^{-12})(1.6\times10^{-19}\,C)^2}{4\pi\left(8.85\times10^{-12}\frac{C^2 s^2}{m^3 kg}\right)(1.78\times10^{-25}\,kg)\left(3\times10^5\frac{m}{s}\right)^2}$$

$$r \sim 3\times10^{-26}\,m$$

DAMN CLOSE, MUCH MORE SO THAN FOR ATOMS!

f



$\pi r^2 v t n = 1$ FOR A STRONG ENCOUNTER

$$t = \frac{1}{n\pi r^2 v} = \frac{1}{(3.8\times10^4\,m^{-3})\,\pi\,(3\times10^{-26}\,m)^2\,(3\times10^5\frac{m}{s})}$$

$$t = 3\times10^{48}\,s = \underline{10^{33}\text{ YEARS}}$$

THAT'S WHAT IN ASTRONOMY WE CALL A "BLOODY LONG TIME"

EVEN USING THE OTHER WEAK FORCE ESTIMATE, WE GET $\underline{10^{23}\text{ YEARS}} \gg t_{univ}$

→ DARK MATTER PARTICLES NEVER RUN INTO EACH OTHER

LONGER THAN STAR AND GAS INTERACTION TIMESCALES

$\sim 10^2 - 10^3$ YEARS FROM HOMEWORK