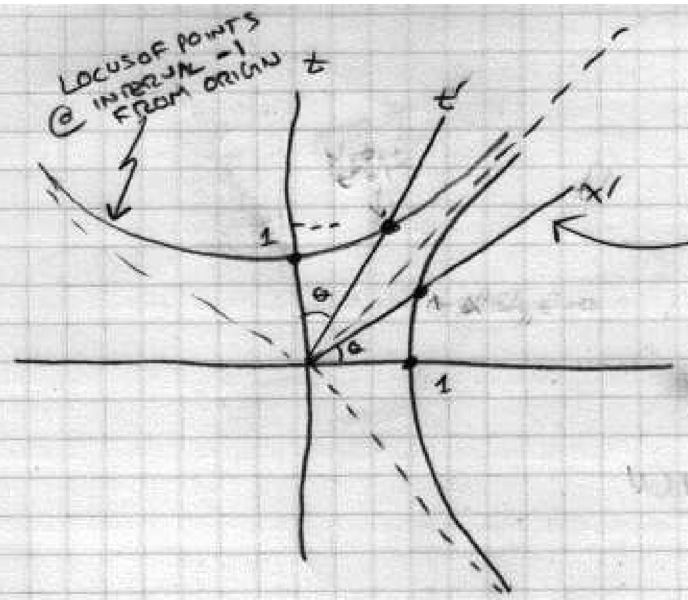
HYPERNOTES 1

2005/02/07



LOCUS OF POINTS AT INTERVAL +1 FROM ORIGIN

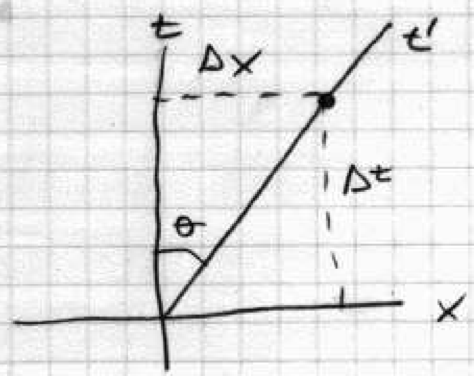
OBSERVER $\mathcal{O}$

$\mathcal{O}'$ $\rightarrow v$

1

CONSIDER WORLD LINE OF $\mathcal{O}'$ IN $\mathcal{O}$ FRAME

↳ THAT'S THE t' AXIS



THIS IS SPEED OF $\mathcal{O}'$ IN $\mathcal{O}$ FRAME

$$\tanh\theta = \frac{\Delta x}{\Delta t} = v$$

$$\boxed{\tanh\theta = v}$$ ⟸ USEFUL!

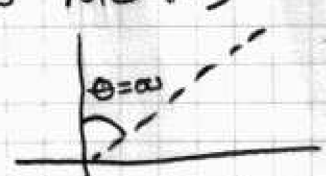
2

HYPERBOLIC IDENTITIES

$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2} \qquad \cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2} \qquad \tanh\theta = \frac{e^{+\theta} - e^{-\theta}}{e^{+\theta} + e^{-\theta}} = \frac{\sinh\theta}{\cosh\theta}$$

As $\theta \to \infty$, $\tanh\theta \to \frac{e^{\theta}}{e^{\theta}} = 1$

$\theta \to \infty$ MEANS $v \to 1$



THESE ANGLES CAN'T BE MEASURED WITH A PROTRACTOR!

⟹

HYPERNOTES 2

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MORE HYPERBOLIC IDENTITIES

$$\cosh^2\theta - \sinh^2\theta = \frac{e^{2\theta} + 2e^{\theta}e^{-\theta} + e^{-2\theta}}{4} - \frac{e^{2\theta} - 2e^{\theta}e^{-\theta} + e^{-2\theta}}{4}$$

NOTE: $e^{\theta}e^{-\theta} = e^{\theta-\theta} = e^{0} = 1$

$$= \frac{2+2}{4} = 1$$

$$\boxed{\cosh^2 - \sinh^2 = 1}$$

$$1 - \tanh^2\theta = 1 - \frac{\sinh^2\theta}{\cosh^2\theta} = \frac{\cosh^2\theta - \sinh^2\theta}{\cosh^2\theta} = \frac{1}{\cosh^2\theta}$$

$$\cosh^2\theta = \frac{1}{1-\tanh^2\theta} = \frac{1}{1-v^2} = \gamma^2$$

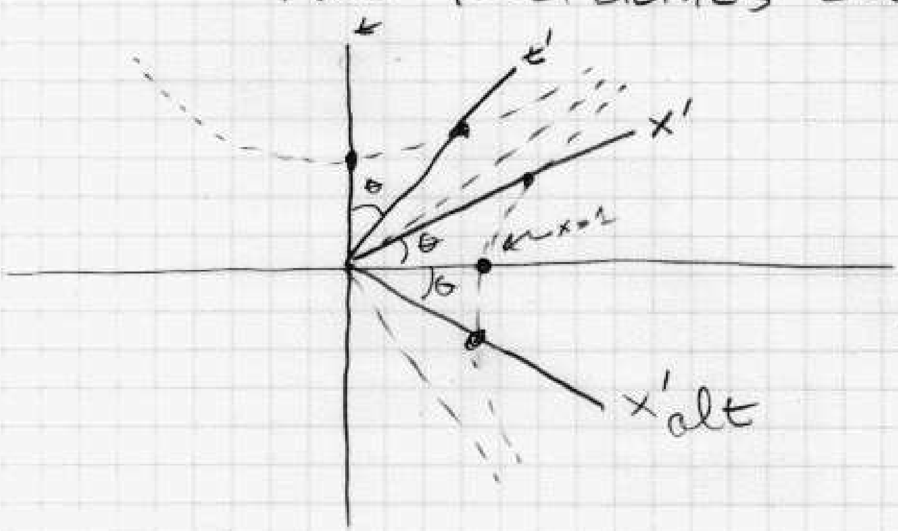
$$\boxed{\cosh^2\theta = \gamma}$$

$$\boxed{1 - \tanh^2\theta = \frac{1}{\cosh^2\theta} = \operatorname{sech}^2\theta}$$

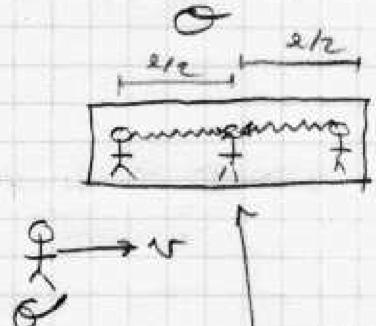
USEFUL!

3 IN A LORENTZ BOOST, WHICH WAY TO DRAW THE X-AXIS ?

TWO POSSIBILITIES SUGGEST THEMSELVES



CONSIDER A TRAIN CAR IN FRAME θ



IF PHOTONS RECEIVED SIMULTANEOUSLY THEN THEY ARE EMITTED SIMULTANEOUSLY

$\Longrightarrow$

HYPERNOTES 3

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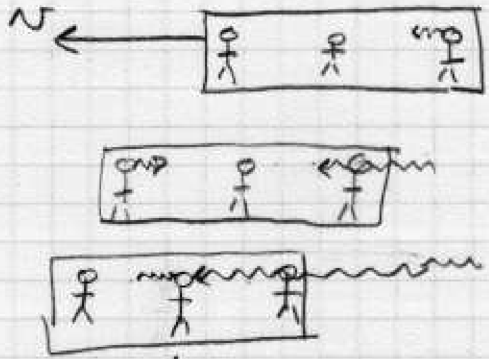
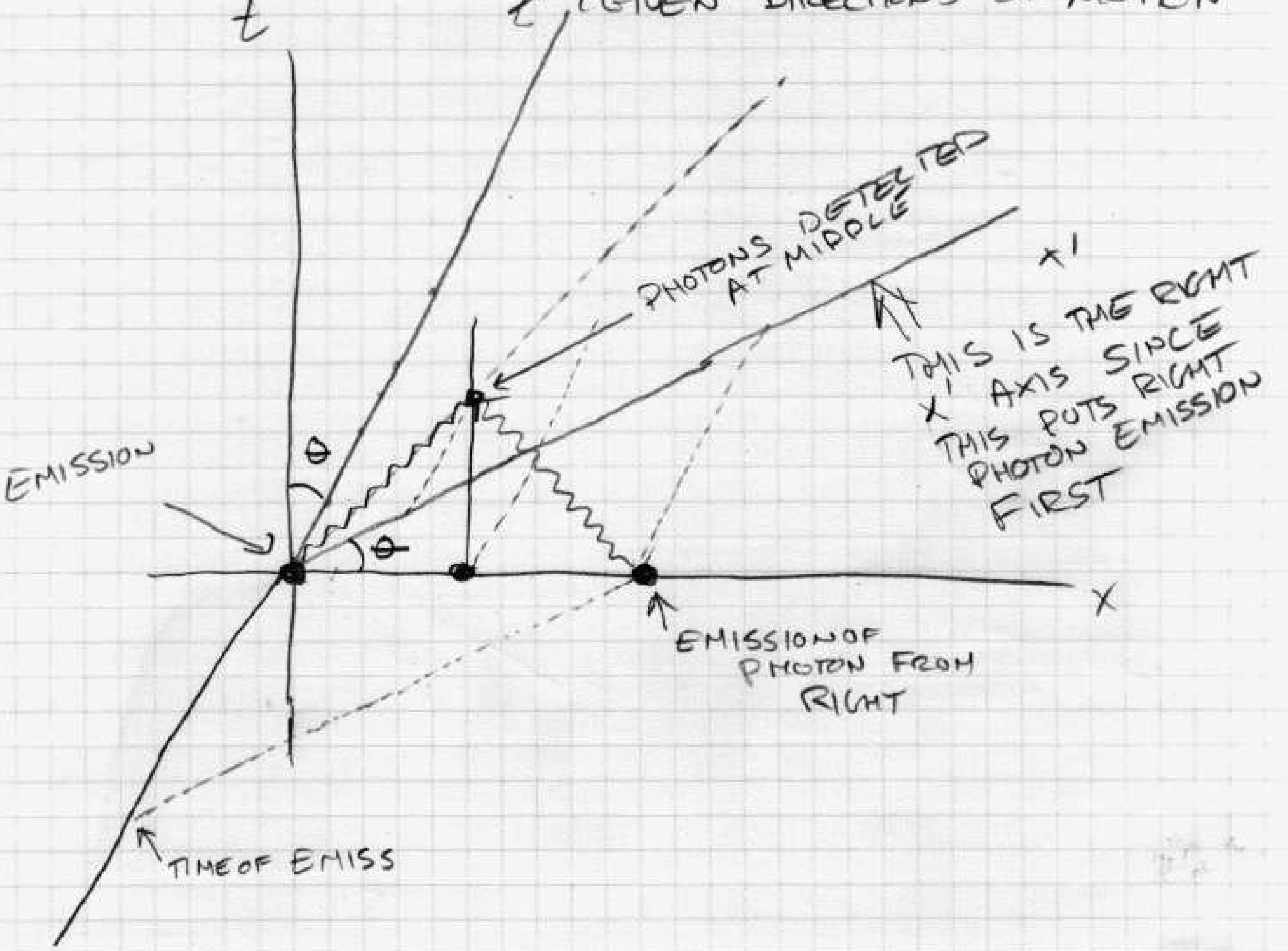
LOOK AT THIS IN THE O' FRAME

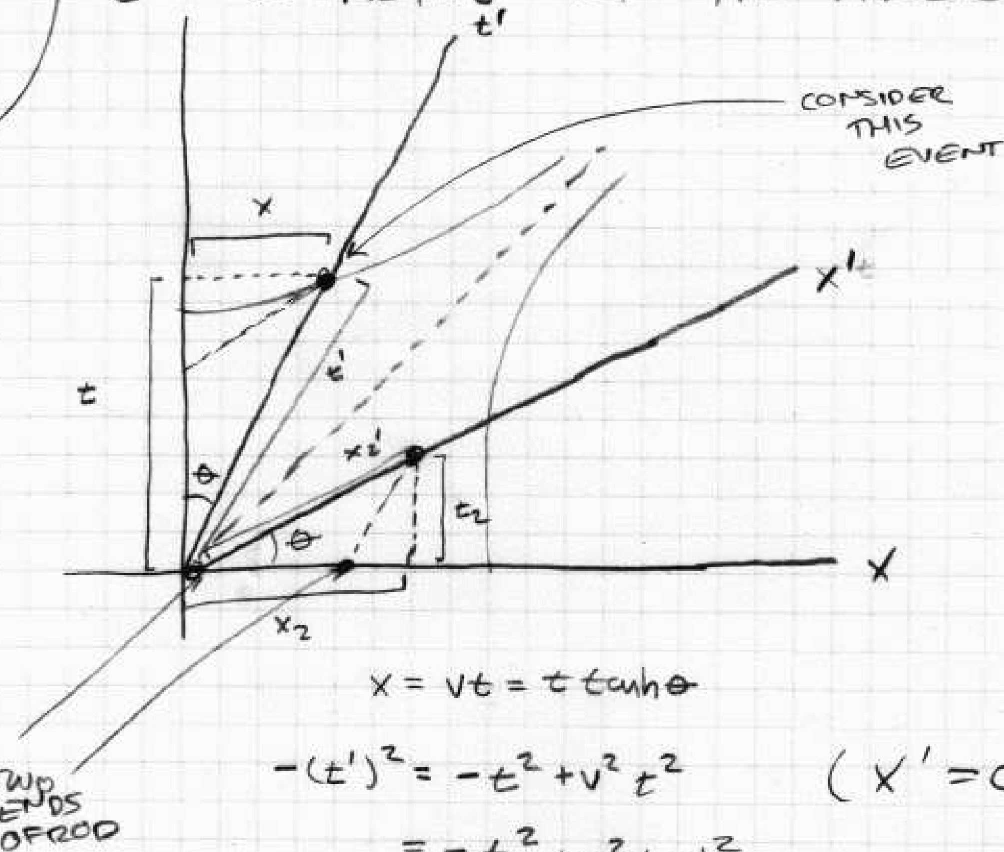


↳ IF PHOTONS RECEIVED SIMULTANEOUSLY, PHOTON AT <u>LARGER X</u> (IN TRAIN FRAME) MUST BE AT <u>EARLIER t'</u> (GIVEN DIRECTIONS OF MOTION



4

SCALING OF THE AXES



$$x = vt = t\tanh\theta$$

$$-(t')^2 = -t^2 + v^2 t^2$$ ($x' = 0 \Rightarrow$ SPECIAL CASE HERE!)

$$= -t^2 + t^2\tanh^2\theta$$

$$-(t')^2 = t^2(1-\tanh\theta)$$

$$t^2 = \frac{(t')^2}{1-\tanh^2\theta} = \gamma^2 (t')^2$$

$$\underline{t = \gamma t'} = (\cosh\theta)\, t'$$ (TIME DILATION)

SO t IS <u>LONGER</u> DESPITE HOW IT LOOKS ON THE EUCLIDEAN PAGE

NOW CONSIDER THE POINT $(x_2', t_2'=0)$

$$\frac{t_2}{x_2} = \tanh\theta$$

$$t_2 = x_2\tanh\theta$$

$$(x_2')^2 = x_2^2 - t_2^2 = x_2^2(1-\tanh^2\theta) = \frac{x_2^2}{\cosh^2\theta} = \frac{x_2^2}{\gamma^2}$$

$$\underline{x_2 = \gamma x_2'}$$

THIS IS <u>NOT</u> LORENTZ CONTRACTION! SEE ENDS OF MOVING ROD ABOVE

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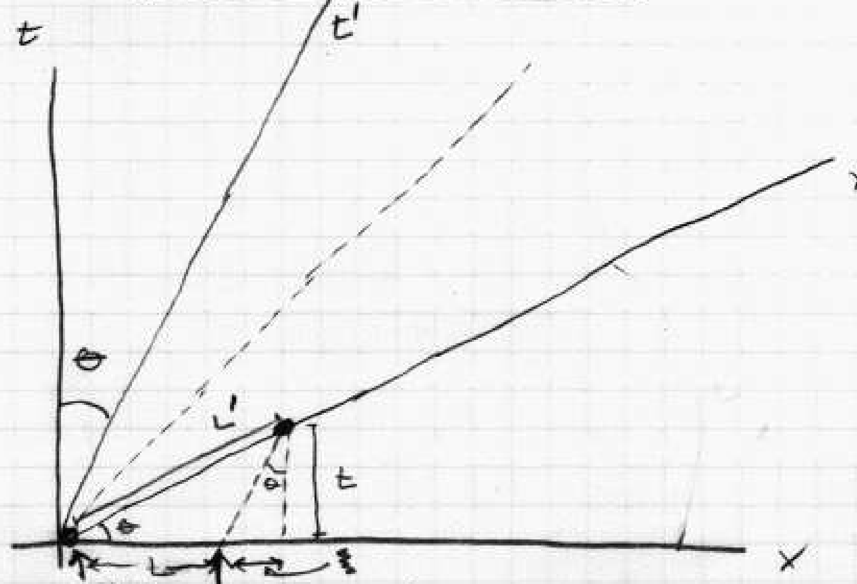
4 cont'd



TWO ENDS OF MOVING ROD, PROPER LENGTH L'

$$\frac{t}{L+\xi} = \tanh\theta \qquad \frac{\xi}{t} = \tanh\theta$$

$$\frac{\xi}{L+\xi} = \tanh^2\theta \qquad t = \frac{\xi}{\tanh\theta}$$

WE ALSO HAVE $(L') = (L+\xi)^2 - t^2$ BY CONSTANCY OF INTERVAL

AND $(L+\xi) = x = \gamma L'$ FROM THE PREVIOUS PAGE

$$\xi = (L+\xi)\tanh^2\theta$$

$$\xi(1-\tanh^2\theta) = L\tanh^2\theta$$

$$\xi = L\tanh^2\theta\cosh^2\theta$$
$$= L\sinh^2\theta$$

$$L(1+\tanh^2\theta\cosh^2\theta) = \cosh\theta\, L'$$

THIS IS THE THING I COULDN'T PROOVE IN CLASS

$$\frac{L}{L'} = \frac{\cosh\theta}{1+\tanh^2\theta\cosh^2\theta} = \frac{\frac{1}{\cosh\theta}}{\frac{1}{\cosh^2\theta}+\tanh^2\theta} = \frac{1}{\cosh\theta}$$

$\Longrightarrow$ $\boxed{\frac{L}{L'} = \frac{1}{\gamma}}$ L IS SMALLER: LORENTZ CONTRACTION

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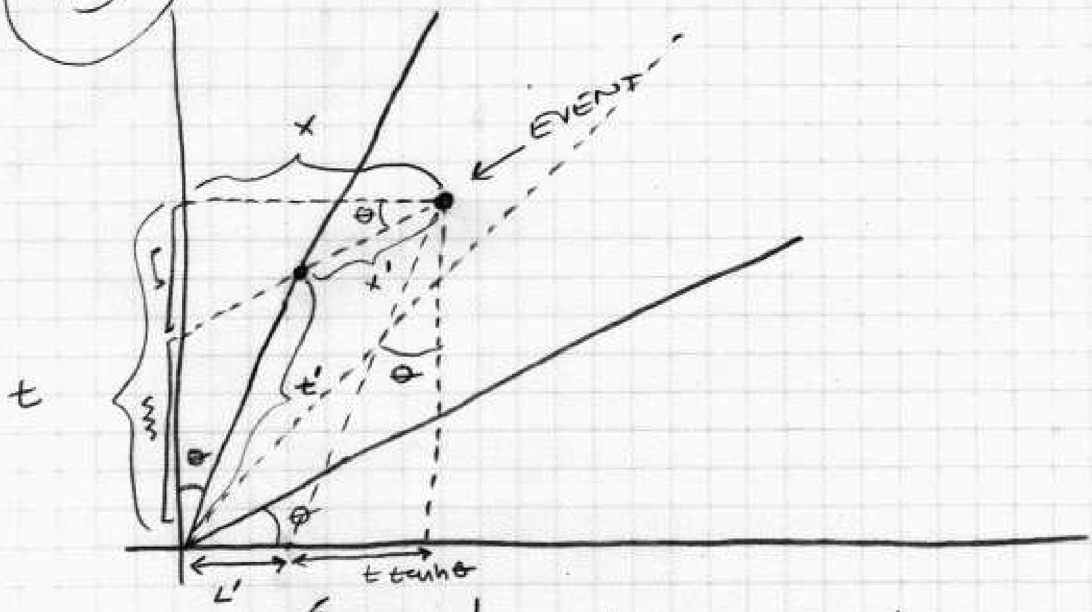
GENERALIZED LORENTZ BOOST



$$\frac{\xi}{t'} = \frac{1}{\cosh\theta} = \frac{1}{\gamma}$$ (JUST LIKE $\frac{L}{L'}$ ON PREV PAGE!)

$$\frac{\zeta}{x} = \tanh\theta$$

$$\xi + \zeta = t$$

$$\frac{t'}{\cosh\theta} + x\tanh\theta = t$$

$$\boxed{t' = \cosh\theta\,(t - x\tanh\theta) = \gamma(t - vx)}$$

$$x = L' + t\tanh\theta$$

$$x = \frac{x'}{\gamma} + t\tanh\theta$$

PREV PAGE

LORENTZ TRANSFORMATION = HYPERBOLIC ROTATION θ

$$\boxed{x' = \cosh\theta\,(x - t\tanh\theta) = \gamma(x - vt)}$$

$$\tanh\theta = v$$