A260 GP 2007-02-20

$\boxed{2}$ (a) m !

$$\underset{\substack{\uparrow \\ \text{OBSERVER}}}{\underline{u}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \underset{\substack{\uparrow \\ \text{OBJECT}}}{\underline{p}} = \begin{bmatrix} m \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(b)

$$-\underline{u} \cdot \underline{p} = m$$

$$\underline{\xi} \cdot \underline{p} = -m$$

$$= g_{\alpha\beta} \xi^\alpha p^\beta = g_{00} \underset{\substack{\uparrow \\ \text{ONLY NON-0} \\ \text{COMP OF } \xi}}{\xi^0} p^0 = (-1)(1)(m) = -m$$

(c)

$$\underline{u} \cdot \underline{u} = -1$$

$$u = \begin{bmatrix} u^0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{u} \cdot \underline{u} = g_{\alpha\beta} u^\alpha u^\beta = g_{00} (u^0)^2 = -1$$

$$u^0 = \sqrt{\frac{-1}{g_{00}}}$$

$$= \left(1 - \frac{2M}{r}\right)^{-1/2}$$

$$= \left(1 - \frac{2M}{6M}\right)^{-1/2} = \sqrt{\frac{3}{2}}$$

$$\boxed{u^\alpha = \begin{bmatrix} \sqrt{\frac{3}{2}} \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

(d)

$$\underline{\xi} \cdot \underline{p} = g_{00} \underset{\uparrow}{\xi^0} p^0 = -m$$

ONLY NON-0 COMP
OF ξ IS ξ^0,
NO OFF-DIAG $g^{\alpha\beta}$

$$-\left(1 - \frac{2M}{r}\right)(1)(p^0) = -m$$

$$p^0 = m\left(1 - \frac{2M}{r}\right)^{-1} = m\left(1 - \frac{2M}{6M}\right)^{-1} = m\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}m$$

$$\boxed{p^0 = \frac{3}{2}m}$$

ONE COULD FIGURE OUT p^1
FROM $\underline{p} \cdot \underline{p} = -m^2$ ⟶

$\boxed{2}$ cont'd

(e)

$$E_{meas} = -\underline{u}\cdot\underline{p}$$
$$= -g_{00}u^0p^0$$
$$= +\left(1-\frac{2M}{6M}\right)\left(\sqrt{\frac{3}{2}}\right)\left(\frac{3m}{2}\right)$$
$$= +\left(\frac{2}{3}\right)\sqrt{\frac{3}{2}}\left(\frac{3}{2}m\right) = \sqrt{\frac{3}{2}}\,m$$

(f)

$$\boxed{\hbar\omega = \left(\sqrt{\frac{3}{2}} - 1\right)m} \qquad (= 0.22m)$$

(g)

@ $r = 6M$, $\underline{u}\cdot\underline{p}_\gamma = -\hbar\omega = \left(\sqrt{\frac{3}{2}}-1\right)m$

$$-g_{00}u^0 p_\gamma^{\,0} = -\left(\sqrt{\frac{3}{2}}-1\right)m$$

$\uparrow$
ONLY NON-0
COMP OF u

$$-\left(1-\frac{2M}{6M}\right)\left(\sqrt{\frac{3}{2}}\right)p_\gamma^{\,0} = -\left(\sqrt{\frac{3}{2}}-1\right)m$$
$$-\left(\frac{2}{3}\right)\left(\sqrt{\frac{3}{2}}\right)p_\gamma^{\,0} = \left(1-\sqrt{\frac{3}{2}}\right)m$$
$$\sqrt{\frac{2}{3}}\,p_\gamma^{\,0} = \left(\sqrt{\frac{3}{2}}-1\right)m$$
$$p_\gamma^{\,0} = \left(\frac{3}{2}-\sqrt{\frac{3}{2}}\right)m$$

$$\underline{p}\cdot\underline{\xi} = g_{00}p^0\xi^0 = -\left(1-\frac{2M}{6M}\right)\left(\frac{3}{2}-\sqrt{\frac{3}{2}}\right)m(1)$$
$$= -\left(\frac{2}{3}\right)\left(\frac{3}{2}-\sqrt{\frac{3}{2}}\right)m$$
$$= -\left(1-\sqrt{\frac{2}{3}}\right)m$$

AT ∞, $\underline{p}\cdot\underline{\xi} = -\left(1-\sqrt{\frac{2}{3}}\right)m$

$$g_{00} = -1$$

SO $p_0 = \left(1-\frac{\sqrt{2}}{\sqrt{3}}\right)m$ $\Rightarrow$

$\boxed{2}$ 9 CONT'D

$$E_{meas} = -\underline{u} \cdot \underline{p} = +p^0 \quad \text{FOR} \quad r = \infty$$

$$\underline{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{\omega_\infty = \left(1 - \sqrt{\tfrac{2}{3}}\right) m}$$

(h) $$\frac{E}{m} = \left(1 - \sqrt{\tfrac{2}{3}}\right) = \boxed{0.18}$$

THAT'S A LOT!
NUCLEAR FUSION
ONLY GIVES ~0.01