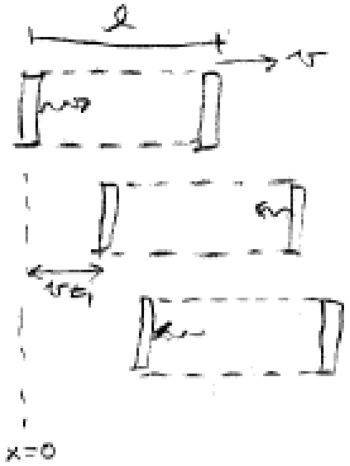
1 a



$t = 0,\ \bar{t} = 0$

REST FRAME:

$c\bar{t}_2 = 2\bar{l}$

$t = t_1$
$\bar{t} = \bar{t}_1$

$t = t_2$
$\bar{t} = \bar{t}_2 = \dfrac{t_2}{\gamma}$ (LESS TIME ELAPSED IN MOVING FRAME)

EQUATION OF MOTION:

$$ct_1 = l + vt_1$$

$$\underbrace{l + vt_1 - c(t_2 - t_1)}_{\text{LIGHT FROM } t_1 \text{ TO } t_2} = \underbrace{vt_1 + v(t_2 - t_1)}_{\text{LEFT SIDE FROM } t_1 \text{ TO } t_2}$$

$$l + vt_1 - ct_2 + ct_1 = vt_2$$

$$(c - v)t_1 = l$$

$$t_1 = \frac{l}{(c-v)}$$

$$l + (v+c)t_1 - ct_2 = vt_2$$

$$l + \frac{(c+v)}{(c-v)}l = (v+c)t_2 \quad t_2 = \gamma\bar{t}_2 = 2\gamma\frac{\bar{l}}{c}$$

$$\frac{cl - vl + cl + vl}{c - v} = (c+v)\,2\gamma\,\bar{l}/c$$

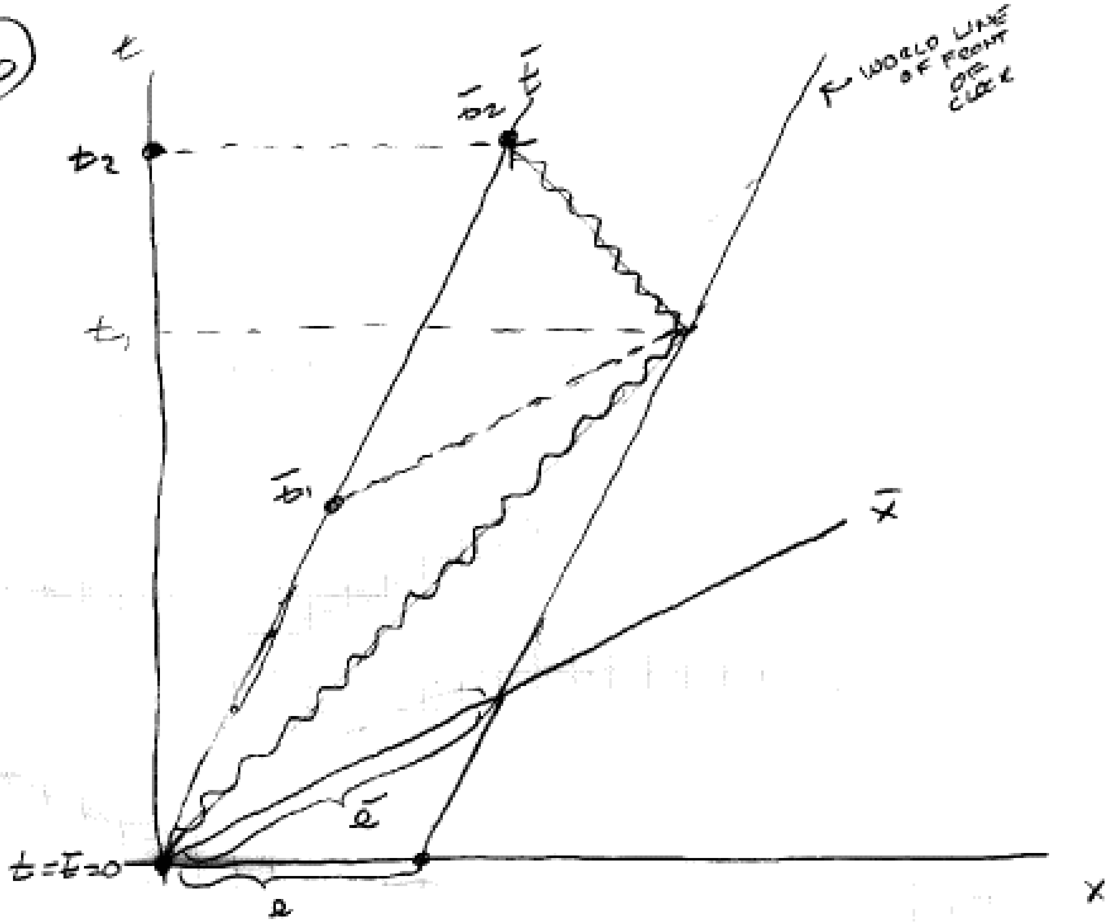
$$\frac{2l}{1 - \frac{v}{c}} = (1 + \frac{v}{c})\,2\gamma\bar{l}$$

$$\frac{l}{1 - \frac{v^2}{c^2}} = \gamma\bar{l}$$

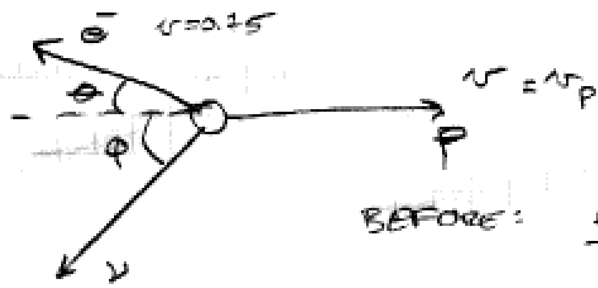
$$\gamma^2 l = \gamma\bar{l}$$

$\boxed{\gamma l = \bar{l}}$ SHORTER IN OUR FRAME THAN IN REST FRAME

(1) (b)



(2)



$m_p = 938.272$ MeV
$m_n = 939.565$ MeV
$m_e = 0.511$ MeV

PARTICLE DATA BOOKLET 2004

$\theta = 24.3^\circ$

SEE GNU OCTAVE CODE NEXT PAGE

BEFORE: $\underline{p} = \begin{bmatrix} m_n \\ 0 \\ 0 \\ 0 \end{bmatrix} =$

AFTER: $\underline{p} = \begin{bmatrix} \gamma_p m_p \\ \gamma_p m_p v_p \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \gamma_e m_e \\ -\gamma_e m_e v_e \cos\theta \\ \gamma_e m_e v_e \sin\theta \\ 0 \end{bmatrix} + \begin{bmatrix} E_\nu \\ -E_\nu \cos\phi \\ -E_\nu \sin\phi \\ 0 \end{bmatrix}$

UNKNOWN: ϕ, v_p, E_ν
γ_p FROM v_p!

SOLVE THE ALGEBRA TO GET
$\phi = 27.3^\circ$
$E_\nu = 0.52$ MeV
$v_p = 0.0011$

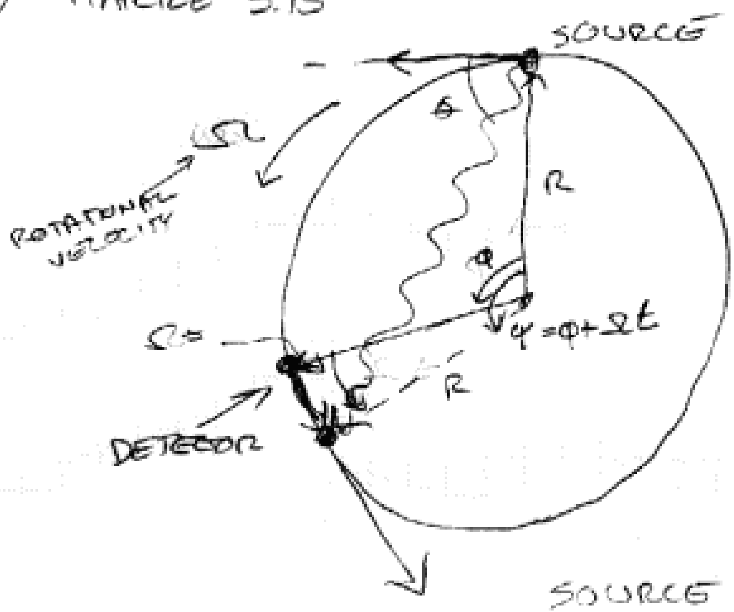
$$m_n = \gamma_p m_p + \gamma_e m_e + E_\nu$$

$$\gamma_e m_e v_e \sin\theta = E_\nu \sin\phi$$

$$\gamma_p m_p v_p = \gamma_e m_e v_e \cos\theta + E_\nu \cos\phi$$

```
# Astro 260, Spring 2007; Homework set 1, question 2
# It's a bit silly to solve these numerically since they can be
#   solve analytically, but this is a good approach for the lazy.
#
# Run this in GNU Octave with "source(hw1-2.oct)"

1;

# x(1) = phi, x(2) = vp, x(3) = Enu


global mn=939.565;
global mp=938.272;
global me=0.511;
global ve=0.75;
global th=24.3*pi/180;

function g=gam(v)
  g=1/sqrt(1-v^2);
endfunction

function y=funcs(x)
  global mn;
  global mp;
  global me;
  global ve;
  global th;
  y(1) = mn - mp*gam(x(2)) - me*gam(ve) - x(3);
  y(2) = gam(ve)*me*ve*sin(th) - x(3)*sin(x(1));
  y(3) = gam(x(2))*mp*x(2) - gam(ve)*me*ve*cos(th) - x(3)*cos(x(1));
endfunction

[x,info,msg] = fsolve("funcs",[pi/4; 0.01; 0.4])
```

(3) HARTLE 5.15



$$\varphi + 2(90^\circ - \theta) = 180^\circ \qquad \text{ISOSCELES}$$

$$\underline{\psi = 2\theta}$$

SOURCE $\underline{u}$ @ EMISSION $= \begin{bmatrix} \gamma \\ -\gamma v \\ 0 \\ 0 \end{bmatrix}$

DETECTOR $\underline{u}$ @ DETECTION $= \begin{bmatrix} \gamma \\ -\gamma v \cos\psi \\ -\gamma v \sin\psi \\ 0 \end{bmatrix}$

PHOTON $\underline{p} = \begin{bmatrix} \hbar\omega \\ -\hbar\omega\cos\theta \\ -\hbar\omega\sin\theta \\ 0 \end{bmatrix}$ NOT ω_*! ω IS IN "OUR" FRAME

$E_{\text{EMITTED}} = \hbar\omega_* = -\underline{u}_{\text{SOURCE}} \cdot \underline{p}$

$= +\gamma\hbar\omega - \gamma v \hbar\omega\cos\theta$

$= \gamma\hbar\omega(1 - v\cos\theta)$

$E_{\text{DETECTED}} = -\underline{u}_{\text{DETECTOR}} \cdot \underline{p}$

$= +\gamma\hbar\omega - \hbar\omega\cos\theta\,\gamma v\cos\psi - \hbar\omega\sin\theta\,\gamma v\sin\psi$

$= \gamma\hbar\omega(1 - v\cos\theta\cos\psi - v\sin\theta\sin\psi)$

$\cos\psi = \cos 2\theta = \cos^2\psi - \sin^2\psi$

$\sin\psi = \sin 2\theta = 2\sin\psi\cos\psi$

$\Rightarrow$

3 CONT'D

$$E_{EMIT} = \gamma \hbar \omega (1 - v \cos\theta)$$

$$E_{DETECT} = \gamma \hbar \omega \left(1 - v\cos\theta(\cos^2\theta - \sin^2\theta) - v\sin\theta(2\sin\theta\cos\theta)\right)$$

$$= \gamma \hbar \omega (1 - v\cos^3\theta + v\cos\theta\sin^2\theta - 2v\sin^2\theta\cos\theta)$$

$$= \gamma \hbar \omega (1 - v\cos^3\theta - v\cos\theta\sin^2\theta)$$

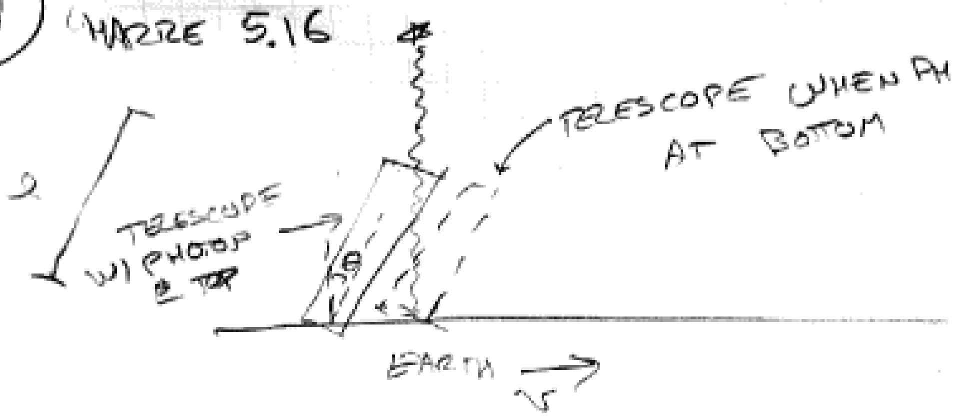
$$= \gamma \hbar \omega (1 - v\cos\theta(\cos^2\theta + \sin^2\theta))$$

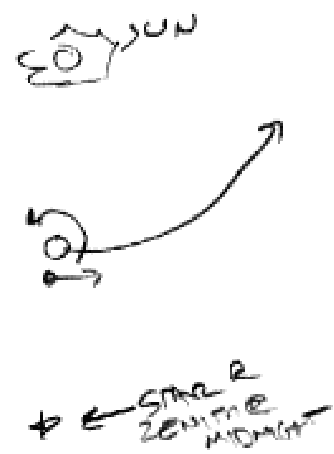
$$= \gamma \hbar \omega (1 - v\cos\theta)$$

$$\boxed{E_{DETECT} = E_{EMIT}}$$

NO SURPRISE ⇒ DISTANCE BET. SOURCE & DETECTION POINT ISN'T CHANGING, SO THERE IS NO RELATIVE VELOCITY!

4 HARRE 5.16





$$l\sin\theta = vt$$

$$l\cos\theta = ct$$

$$\boxed{\tan\theta = \frac{v}{c}}$$

(4) CONT'D

$$v_{ORBIT} = \frac{2\pi(1\,AU)}{1\,year} = 2\pi\,\frac{AU}{year} = 29{,}800\,\frac{m}{s}$$

$$= 9.94\times10^{-5}\,c$$

$$v_{ROT} = \frac{2R_\oplus}{24\text{ hours}} = \frac{2(6378\,km)}{24\text{ hours}} = 148\,\frac{m}{s}$$

$$= 4.9\times10^{-7}\,c$$

$$v_{ORBIT} + v_{ROT} = 9.99\times10^{-5}\,c$$

ADD NONRELATIVISTICALLY 'CAUSE THEY'RE SMALL

$$\frac{v}{c} = 10^{-4}$$

$$\tan\theta = 10^{-4} \rightarrow \theta = 10^{-4}\text{ RADIANS}$$

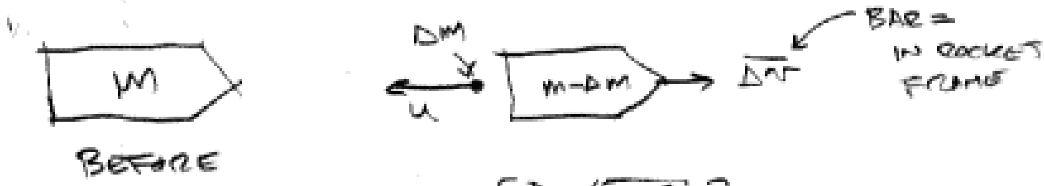
$$= 0.006^\circ$$

$$= \underline{\underline{20\text{ ARCSECONDS}}}$$

TO AN ASTRONOMER, THIS IS BIG

(5) MAR RE 5.22

IN MCRF OF ROCKET



$$P = \begin{bmatrix} m \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad P_{MASS} = \begin{bmatrix} \Delta m/\sqrt{1-u^2} \\ -u\,\Delta m/\sqrt{1-u^2} \\ 0 \\ 0 \end{bmatrix}$$

$$P_{ROCKET} = \begin{bmatrix} (m-\Delta m)/\sqrt{1-\overline{\Delta v}^2} \\ (m-\Delta m)\overline{\Delta v}/\sqrt{1-\overline{\Delta v}^2} \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$

(5) CONT'D

CONSERVE:

$$\frac{m-\Delta m}{\sqrt{1-\Delta v^2}} + \frac{\Delta m}{\sqrt{1-u^2}} = m$$

$$\frac{u\,\Delta m}{\sqrt{1-u^2}} = \frac{(m-\Delta m)\,\overline{\Delta v}}{\sqrt{1-\overline{\Delta v}^2}}$$

$$\frac{\overline{\Delta v}}{\sqrt{1-\overline{\Delta v}^2}} = \frac{u\,\Delta m}{(m-\Delta m)\sqrt{1-u^2}}$$

$m-\Delta m = m\left(1-\frac{\Delta m}{m}\right)$

Δv is SMALL, Δm is SMALL

GET ALL $\overline{\Delta v}$'S AND Δm'S ON THE TOP SO WE CAN EASILY SEE WHAT IS DOUBLE DINKY

$$\overline{\Delta v}\left(1+\tfrac{1}{2}\underbrace{\overline{\Delta v}^2}_{\substack{\text{DINKY}\\(\Delta v^3)}}\right) = \frac{u\,\Delta m}{m}\left(1+\underbrace{\frac{\Delta m}{m}}_{\substack{\text{DINKY}\\(\Delta m^2)}}\right)\frac{1}{\sqrt{1-u^2}}$$

+ FROM BINOMIAL THM

$$\overline{\Delta v} = \frac{u\,\Delta m}{m\sqrt{1-u^2}} \qquad \text{OR} \qquad \Delta m = \frac{m\,\overline{\Delta v}\sqrt{1-u^2}}{u}$$

THIS IS ALL WE NEED!

TRANSLATE TO OUR FRAME

IN MCRF OF ROCKET

$$\Delta \underline{u}_{\text{ROCKET}} = \begin{bmatrix} 1/\sqrt{1-\overline{\Delta v}^2} \\ \Delta v/\sqrt{1-\overline{\Delta v}^2} \\ 0 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 1 \\ \overline{\Delta v} \\ 0 \\ 0 \end{bmatrix}$$

$$\Lambda^{\alpha}{}_{\beta} = \begin{bmatrix} \gamma & \gamma v & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Delta \underline{u}_{\text{ROCKET}} = \begin{bmatrix} \gamma + \gamma v\,\overline{\Delta v} \\ \gamma v + \gamma\,\overline{\Delta v} \\ 0 \\ 0 \end{bmatrix} = \underline{u}_{\text{AFTER}}$$

IN OUR FRAME

= AFTER EJECTION OF Δm

(5) cont'd

In our frame $u_{before} = \begin{bmatrix} \gamma \\ \gamma v \\ 0 \\ 0 \end{bmatrix}$

Δv = change in velocity in our frame

$\overline{\Delta v}$ = change in velocity in MCRF of rocket

$$\underline{u}_{after} = \begin{bmatrix} \frac{1}{\sqrt{1-(v+\Delta v)^2}} \\ \frac{v+\Delta v}{\sqrt{1-(v+\Delta v)^2}} \\ 0 \\ 0 \end{bmatrix}$$

⇐ Also $\underline{u}$ of rocket after ejection of Δm, components in our frame

This is the same set of components! Thus, we can set them equal

$$\underline{u}_{after}^0 = \gamma + \gamma v \overline{\Delta v} = \frac{1}{\sqrt{1-(v+\Delta v)^2}}$$

$$\underline{u}_{after}^1 = \gamma v + \gamma \overline{\Delta v} = \frac{v+\Delta v}{\sqrt{1-(v+\Delta v)^2}}$$

$$\frac{v+\overline{\Delta v}}{\sqrt{1-v^2}} \simeq \frac{v+\Delta v}{\sqrt{1-v^2-2v\Delta v}} = \frac{v+\Delta v}{\sqrt{1-v^2}\sqrt{1-\frac{2v\Delta v}{1-v^2}}}$$

Binomial

$$\simeq \frac{v+\Delta v}{\sqrt{1-v^2}}\left(1+\frac{v\Delta v}{1-v^2}\right)$$

$$\cancel{\frac{v}{\sqrt{1-v^2}}} + \frac{\overline{\Delta v}}{\sqrt{1-v^2}} \simeq \cancel{\frac{v}{\sqrt{1-v^2}}} + \frac{\Delta v}{\sqrt{1-v^2}} + \frac{v^2\Delta v}{1-v^2}$$

(dropping Δv^2 terms)

$$\frac{\overline{\Delta v}}{\sqrt{1-v^2}} = \Delta v\left(\frac{1}{\sqrt{1-v^2}} + \frac{v^2}{1-v^2}\right)$$

$$\frac{\overline{\Delta v}}{\sqrt{1-v^2}} = \Delta v\left(\frac{\sqrt{1-v^2}+v^2}{1-v^2}\right)$$

$$\Delta v = \frac{\sqrt{1-v^2}}{\sqrt{1-v^2}+v^2}\overline{\Delta v}$$

Check: As $v \to 1$, $\Delta v \to 0$
As $v \to 0$, $\Delta v \to \overline{\Delta v}$

(5) CONT'D

SUBSTITUTE FOR Δv

$$\Delta v = \frac{\sqrt{1-v^2}}{\sqrt{1-v^2}+v^2} \frac{u\,\Delta m}{m\sqrt{1-u^2}}$$

$$\frac{\sqrt{1-v^2}+v^2}{\sqrt{1-v^2}} \Delta v = \frac{-u}{\sqrt{1-u^2}} \frac{\Delta m}{m}$$

I JUST STUCK THIS IN 'CAUSE $v \uparrow$ AS $m \downarrow$ ....

RANGE OF VEL

$$\int_0^{V_f} \frac{\sqrt{1-v^2}+v^2}{\sqrt{1-v^2}}\, dv = \frac{-u}{\sqrt{1-u^2}} \underbrace{\int_{M_i}^{M_f} \frac{dm}{m}}_{\text{THIS IS EASY}}$$

$$\int_0^{V_f} \left(1 + \frac{v^2}{\sqrt{1-v^2}}\right) dv = -\frac{u}{\sqrt{1-u^2}} \ln \frac{M_f}{M_i}$$

(>0 FOR $M_f < M_i$)

$$V_f + \underbrace{\int_0^{v_f} \frac{v^2}{\sqrt{1-v^2}}\, dv}_{} =$$

I CHEATED AND USED MAXIMA (IF ONE WERE CLEVER, A CHANGE OF VARIABLES WOULD DO IT)

$$V_f + \ln\left(\frac{1+V_f}{1-V_f}\right) - V_f = \frac{-u}{\sqrt{1-u^2}} \ln \frac{M_f}{M_i}$$

$$\ln\left(\frac{1-V_f}{1+V_f}\right)\left(\frac{\sqrt{1-u^2}}{u}\right) = \ln \frac{M_f}{M_i}$$

$$\left(\frac{1-V_f}{1+V_f}\right)^{\frac{\sqrt{1-u^2}}{u}} = \frac{M_f}{M_i}$$

For $V_f = 0.5$, $u = 0.9$, $\frac{M_f}{M_i} = 0.79$

For $V_f = 0.1$, $u = 0.1$, $\frac{M_f}{M_i} = 0.14$ DDD