A311 P5

— FOR IONIZED GAS —

1. The heating mainly from free electrons produced by the photoionization of Hydrogen, which is identical in both cases. The free e^- then are able to collisionally excite heavy metals (for which ionization potentials are much higher) which then radiatively de-excite sometimes. Since radiative excitation cross-sections are low, this radiation can escape. This energy ~~[illegible]~~ came originally from the colliding free e^-, hence the gas temperature is lowered (cooling).

2. Cloud with $\frac{10^{10} M_\odot\ H^\circ}{\text{neutral gas}}$ (ionization fraction 10^{-4})
$n = 1\ cm^{-3}$

if heating only from kinetic energy from Type II SNe (10^{51} erg per SN) which occur at a rate of 1 per 100 yr

What is the equilibrium T_{gas}?

with $\zeta = 10^{-4}$, assume $n = 1 cm^{-3}$ refers to H°, also assume negligible metal content for this number.

Use Spitzer Fig 6.2 cooling rates

- At equilibrium, heating rate = cooling rate so need to get rid of

$$\frac{10^{51}\,erg}{10^2 yr \times 3.17\times 10^7 s/yr} = \boxed{3.15\times 10^{41}\ erg\ s^{-1}}\ \text{over entire cloud.}$$

$$\text{Volume of } H^\circ = \frac{10^{10} M_\odot \cdot 2\times 10^{33} g/M_\odot}{1 cm^{-3} \times 1.67\times 10^{-24} g/H^\circ} = \boxed{1.2\times 10^{67}\ cm^3}$$

$$\text{Then need to cool at rate of} = \frac{3.15\times 10^{41}\ erg\,s^{-1}}{1.2\times 10^{67} cm^3} \times \frac{1}{(1 cm^{-3})^2} = \boxed{2.63\times 10^{-26}\ \frac{erg}{cm^3 s^{-1}}}$$

gas Temp from Fig 6.2: $\boxed{T \sim 10^{3.5} = 3\times 10^3 K}$

3 (a) $\frac{\partial u_i}{\partial x_k} = 0$ EXCEPT FOR $\frac{\partial u_x}{\partial x_y}$

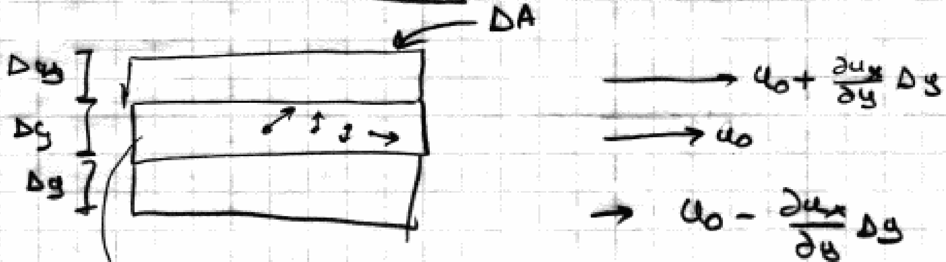
THUS

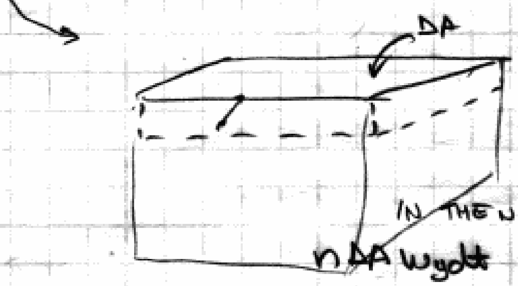
$$\boxed{D_{xy} = D_{yx} = \frac{\partial u_x}{\partial x_y} \qquad \text{ALL OTHER } D_{ij} = 0}$$

(b) $\mu \vec{\nabla} \cdot \overleftrightarrow{D} = \mu \frac{\partial}{\partial x_i} D_{ij} \Rightarrow$ VISCOUS FORCE $= \mu D_{ij}$

$$\boxed{\text{VISCOUS FORCE} = \mu \frac{\partial u_x}{\partial x_y}}$$

(c)





$dy = w_y dt$ (dy NOT THE SAME AS Δy!)

IN THE NEXT dt

$n \Delta A w_y dt$ PARTICLES WILL CROSS

$$\frac{1}{2} m \langle w_y^2 \rangle = \frac{1}{2} kT$$

WHAT I MEAN BY "w_y" ABOVE $\longrightarrow \sqrt{\langle w_y^2 \rangle} = \sqrt{\frac{kT}{m}}$

RATE OF PARTICLES CROSSING $= n \Delta A w_y$

$$\boxed{\text{RATE} = n \Delta A \sqrt{\frac{kT}{m}}}$$

(d) TYPICAL MEAN DISTANCE $= \ell$ = MEAN FREE PATH

$$\boxed{\ell = \frac{1}{n\sigma}}$$

(e) IF WE CHOOSE $\Delta y = \ell$, THEN THE AVERAGE X-VELOCITY OF PARTICLES FROM THE FLUID ELEMENT BELOW IS:

$$\bar{u}_x = u_0 - \frac{\partial u_x}{\partial y}\ell = u_0 - \frac{\partial u_x}{\partial y}\frac{1}{n\sigma}$$

$\bar{}$ FOR "BELOW"

THE VELOCITY RELATIVE TO OURS IS

$$-\frac{\partial u_x}{\partial y}\frac{1}{n\sigma}$$

SO THE MOMENTUM TRANSFERRED IS

$$\boxed{-\frac{\partial u_x}{\partial y}\frac{m}{n\sigma} \quad \text{PARTICLES FROM BELOW (EACH ONE)}}$$

SIMILARLY

$$\boxed{+\frac{\partial u_x}{\partial y}\frac{m}{n\sigma} \quad \text{PARTICLES FROM ABOVE (EACH ONE)}}$$

(f) TOTAL VISCOUS FORCE ON ONE FACE =

$$\Delta A\left(\underbrace{\mu\frac{\partial u_x}{\partial y}}_{\text{FORCE / UNIT AREA}}\right) = (\#\ \text{PARTICLES})\left(\frac{\partial u_x}{\partial y}\frac{m}{n\sigma}\right)$$

$$\cancel{\Delta A}\,\mu\cancel{\frac{\partial u_x}{\partial y}} = \cancel{n}\cancel{\Delta A}\sqrt{\frac{kT}{m}}\,\cancel{\frac{\partial u_x}{\partial y}}\,\frac{m}{\cancel{n}\sigma}$$

$$\boxed{\mu = \frac{\sqrt{mkT}}{\sigma}}$$

(TO ORDER OF MAGNITUDE)

(g) NOTE: DENSITY DOES NOT APPEAR IN VISCOUS COEFFICIENT! THIS IS REASONABLE SINCE FOR LOWER DENSITY, A PARTICLE CAN CARRY ITS X-MOMENTUM THROUGH A GREATER VELOCITY GRADIENT, SO MOMENTUM TRANSFER IS MORE FOR EACH PARTICLE. IN HIGH DENSITY, THEY DON'T GO AS FAR BUT THE COLLISION RATE IS HIGHER, MAKING MOMENTUM TRANSFER FASTER.