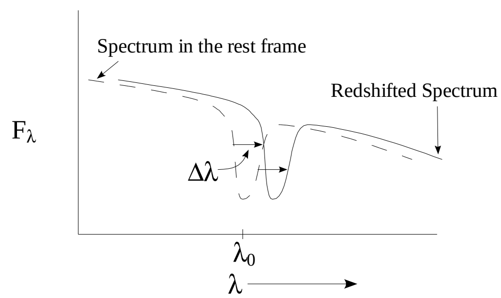
Astronomy 253 — Group Problems 2

Friday, January 30

Do the problems on separate sheets of paper. Each group will turn in only one set of solutions. Make sure your solutions are clear enough that I can understand what you were doing and what you were thinking! You probably won't get through all of these problems in an hour; that is OK. Physics majors should do Problem 5 first; otherwise, do all problems in order.

1. Astronomers can measure the *radial velocity* (that is, velocity towards us or away from us) by measuring the Doppler shift of an object. To do this, we obtain a spectrum of the object. We find spectral features whose normal ("rest") wavelengths we know (e.g. the Balmer lines of Hydrogen, Hα, Hβ, etc). We measure where they appear in the object. If the object is moving *away* from us, the star will be *redshifted*; that is, the lines will appear at longer (redder) wavelengths. Similarly, if an object is moving *towards* us, the star will be *blueshifted.*



The redshift z is defined by

$$z = \frac{\Delta\lambda}{\lambda_0}$$

where λ_0 is the rest wavelength of the feature.

(a) Special Relativity gives us the formula for a Doppler shift:

$$1 + z = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}},$$

where positive v is velocity away from the observer. Show that for $v \ll c$, this reduces to $z = v/c$.

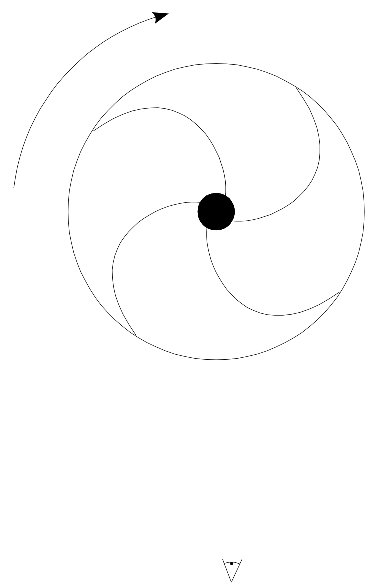
(Non-Physics majors may skip this part, and assume the result.)

(b) You take a spectrum of an A0 star. The $H\alpha$ line (rest wavelength 6563Å) is observed at 6568Å. Is this object moving towards us or away from us? How fast?

(c) Assuming that the Sun orbits the galaxy in a circle, should an object exactly at the Galactic center appear redshifted or blueshifted to us? Explain.

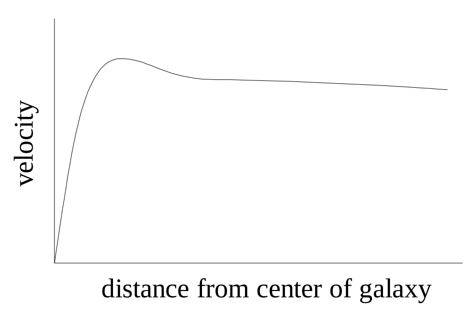
(d) Suppose you're on Mars, which is in an elliptical orbit, and that Mars has recently passed perihelion (the closes point to the Sun). Will you see the spectral lines in the Sun redshifted, blueshifted, or unshifted? Explain.

2. You observe a rotating spiral galaxy edge on:



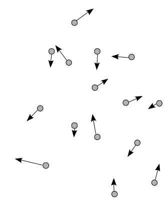
Suppose this galaxy has a rotation curve like a merry-go-round. (Ignore the effect of snot-nosed kids.) Make a plot of the *observed* wavelength of the Hα line vs. observed angular distance from the center of the galaxy. Assume that the galaxy subtends $15'$ on the sky, and that the maximum velocity you observe is 250 km/s.

3. Consider the spiral galaxy of the previous problem. As mentioned in class, galaxies don't really rotate like merry-go-rounds; rather, they have a *rotation curve*, which tells you the *linear* (not angular) velocity of a star or gas cloud at a given distance from the center of the galaxy, that looks something like this:



Sketch a plot that roughly shows the *observed* wavelength of an Hα line vs. observed angular distance from the center of the galaxy, again assuming that the galaxy subtends $15'$ on the sky and that the maximum velocity you observe is 250 km/s.

4.



Imagine an astrophysical gas with Hydrogen atoms moving around every which way. Suppose these atoms are emitting Hα light. For a low density gas with all of the atoms at rest, this light will always be emitted at very close to exactly 6563Å, and you would observe a spectral "emission line" with all of its wavelength at that wavelength. However, if a given atom is moving towards you, the Hα light that one atom emits will be blueshifted; if another is moving away from you, the light emitted by *that* atom will be redshifted.

(a) Suppose that in contrast to what you might expect from statistical mechanics, the line-of-sight (radial) velocity distribution of the atoms is evenly distributed between -100 km/s and 100 km/s (that is, you are equally likely to find a given atom moving toward or away from you at any velocity between 0 and 100 km/s). Sketch what the observed Hα emission line would look like.

(b) Now suppose a more reasonable distribution, a Gaussian (bell-curve) distribution centered around 2,000 km/s, with full width at half maximum (FWHM) of 200 km/s. Sketch the spectral line that you would observe.

(c) The FWHM of a spectral line can sometimes be used to diagnose the temperature of an astrophysical gas. Qualitatively, how would you expect an observed spectral line change as the temperature of a gas changes? (Comment on the width and observed wavelength of the line, as well as on the total energy in the line.)

5. When the Sun leaves the main sequence, it will go into its "red giant" phase, with a degenerate inert Helium core surrounded by a shell of Hydrogen which is fusing to Helium. Helium "ash" builds up in the core, contracting it, until it is dense and hot enough to ignite Helium fusion. At this point, the Helium core has a radius of roughly 7,000 km, and a mass of about 1/3 the mass of the Sun. (1 $M_\odot = 2 \times 10^{3}0$ kg.) The excellent thermal conduction of the degenerate Helium core allows much of the Helium to fuse all at once, resulting in the Helium flash. This fuses (say) a tenth of the Helium up to Carbon, and expands the core out to a radius of about 70,000km, over the course of minutes or hours.

(a) Even though this helium flash happens very quickly, and is a tremendous thermonuclear explosion, no direct effect is seen right away in the luminosity of the star. Where is all the energy from that fusion going?

(b) (Physics Majors Only.) Do an order of magnitude energy calculation to show that your answer to (a) is plausible. Helium is fused to Carbon via the "triple-alpha" process. The mass of one Helium-4 atom is 4.0026032u and the mass of one Carbon-12 atom is 12.0000000u.