Astronomy 253

Review for Exam 2

2004-April-09

1. The Milky Way and Andromeda Galaxies are approaching each other at 120 km s^{-1}.

   (a) Nonetheless, we do not know if they will run into each other at their next encounter. Why don't we know this? Sketch the two galaxies showing qualitatively a couple of possible trajectories, indicating what is measured as 120 km $^{-1}$.

   (b) If the two galaxies are destined for a head-on collision, and they are currently $\sim$ 750 kpc apart, how many years will it be before they run into each other (estimating roughly)?

   (c) If the galaxies do miss each other the next time they pass, and if we ignore the gravitational influence of all other members of the local group, is it impossible, unlikely, possible, likely, or inevitable that the two galaxies will still eventually merge? Why or why not?

2. Describe in a few sentences (possibly with diagrams) *two* ways of estimating the mass of a cluster of galaxies. Be sure to explain the physical concept behind each method.

3. Consider a spherically symmetric galaxy with density profile $\rho = \rho_0$ for $r \leq a$ and $\rho = 0$ for $r > a$.

   (a) What is the total mass $\mathcal{M}$ of this galaxy?

   (b) What is the expression for the potential Φ for $r > a$?

   (c) Show that the potential for $r < a$ is

   $$\Phi(r) = \frac{2\pi G\rho_0}{3}r^2 + \text{const}$$

   What is the value of the constant (and why)? (Note: try to do this without using the argument "it is because equation 3.33 says so".)

   (d) What is the total potential energy $\mathcal{PE}$ of this galaxy?

   (NOTE: we have not talked about this in class, although it was in the reading. We did mention that the potential energy of one particle in potential Φ is $m\,\Phi$. If you think about adding up all of the particles together, you'll get:

   $$\mathcal{PE} = \frac{1}{2}\int \rho(\vec{r})\Phi(\vec{r})d^3r = -\frac{1}{8\pi G}\int \left|\vec{\nabla}\Phi(\vec{r})\right|^2 d^3r$$

   where the integrals are over all space. In the first expression, $\rho(\vec{r})d^3r$ is just like m, i.e. the mass in little volume d^3r. The integral is just adding all of that up. The 1/2 comes from the fact that in so doing, you have double counted everything, since you counted the m not only when it was interacting with Φ, but also when it was helping create Φ for all the other little m's. The second term may be derived from Poisson's equation and a little vector magic, about which you should not worry much.

An integral over all space:

$$\int f(\vec{r})\ d^3r$$

can also be written

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} f(\vec{r})\ r^2 \sin\theta\ dr\ d\theta\ d\phi$$

which, with spherical symmetry, reduces to

$$\int_0^{\infty} f(r)\ 4\pi r^2\ dr$$

)

(e) What is $v_c(r)$, the circular velocity at radius r?

(f) Assuming there are no random motions (i.e. everything is orbiting circularly around the center of the galaxy), is this galaxy in equilibrium?

4. This one is hard. Harder than what I'll give you on the test, although the same concepts are tested. But it's kinda cool, I think.

The mass of stars in the Galaxy is about 10^11 $M_\odot$. The mass of dark matter is about ten times that. In the solar neighborhood, there is approximately one star of mass 1 $M_\odot$ every 10 pc^3. The dark matter is (probably) distributed roughly in a sphere of radius $\sim$ 100 kpc, whose density drops off with distance. In the solar neighborhood, the density of dark matter is probably roughly similar to the density of stars (as we're in a very star-rich area).

(a) Suppose that each dark matter particle is a new fundamental particle that is quite massive (for a fundamental particle), with $\mathrm{m\,c^2} = 100\,\mathrm{GeV}$. What is the mass of each dark matter particle in kg?

(b) What is the *number density* (particles per cubic meter, or cubic other-length-unit) of dark matter?

(c) What, very very roughly, is the potential energy of the dark matter halo?

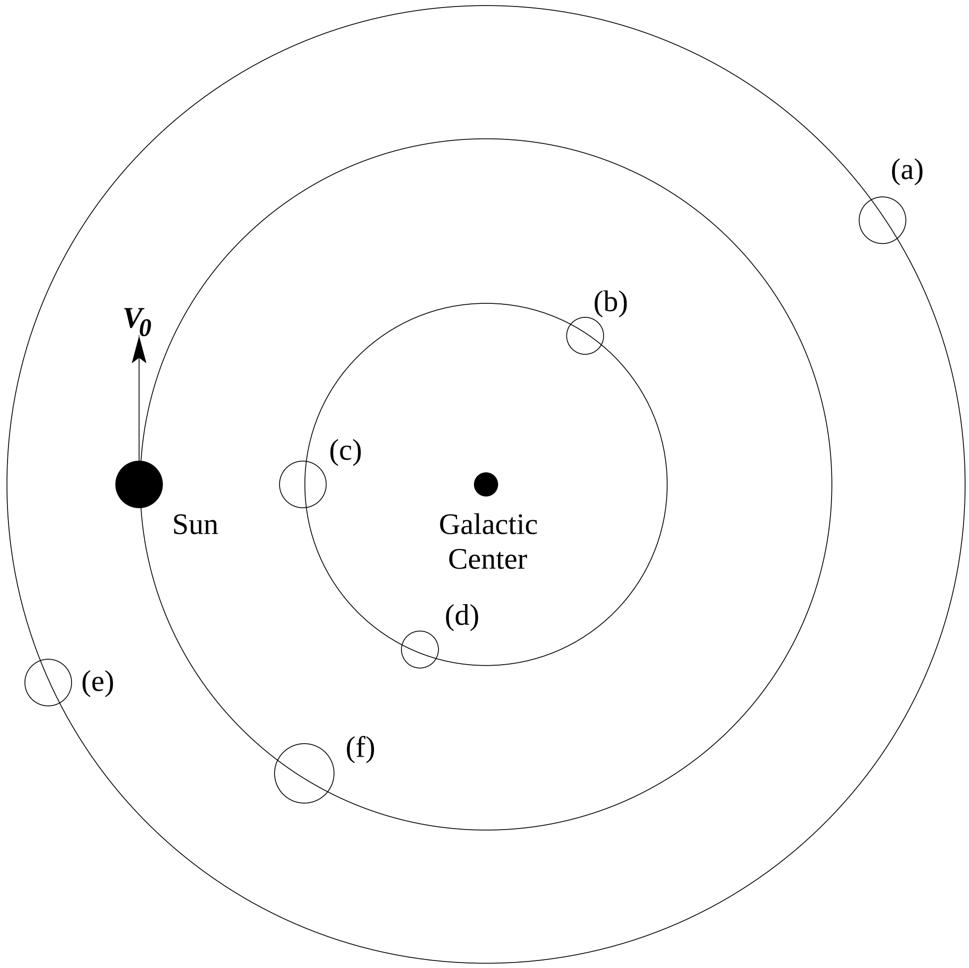
(d) Assuming the dark matter halo to be in virial equilibrium, what is the average velocity of each dark matter particle?

(e) Whereas stars interact mainly via gravity, atoms interact via the electromagnetic force once they get close enough for their electron clouds to mingle; even at these distances, gravity is unimportant. Dark matter particles probably interact via the "weak force", which is $\sim 10^{25}$ times stronger than gravity, but only $\sim 10^{-12}$ as strong as the electromagnetic force. Use just this information to make a very rough estimation of ow close must two dark matter particles get for them to have a "strong" interaction. (That is, for the potential energy to be of the same order of magnitude as the kinetic energy.)

(f) What is the timescale for strong interactions of dark matter particles? How does this compare to the timescales for stars, and for atoms of Hydrogen gas?

(g) When two galaxies run into each other, which components of the galaxy are likely to be affected by anything other than gravity?

5. Consider the following drawing of the Galactic plane, looking down from Galactic North:



The position and velocity of the Sun is indicated. Assuming that the Galaxy has a flat rotation curve $v_c(R) = V_0$, indicate for each blob of gas (a) through (f) whether an observed spectral line from that blob will be *redshifted*, *blueshifted*, or *unshifted* relative to its normal wavelength.

(a)	redshifted	blueshifted	unshifted
(b)	redshifted	blueshifted	unshifted
(c)	redshifted	blueshifted	unshifted
(d)	redshifted	blueshifted	unshifted
(e)	redshifted	blueshifted	unshifted
(f)	redshifted	blueshifted	unshifted