A311 Final

This and the Solo Section of the final are both due Friday, December 16

This part of the final is to be completed in collaboration with your assigned group. Each group should turn in *one* set of solutions, and everybody in the group will receive the same grade on that part of the final.

Do not look at the solo section of the final until you have turned in this section.

You are allowed to use Spitzer, Osterbrock, a table of physical constants (such as the Particle Data Booklet, or a scientific calculator with constants and the ability to convert units), the knowledge that 1 eV = 1.60×10^{-12} ergs, your class notes, and any problem sets and solutions I have posted on the course web page. For the first problem, you may use your numerical problem solution and all the various computer code that supports it. Do not use any other references during this final.

1. The ionizing continuum that creates regions of ionized gas is sometimes better described by a power law than it is by the single-temperature blackbody found around a hot young star— for example, in the ionizing spectrum produced by the black hole accretion disk in an AGN. Although the UV continuum is really more complicated than this (it gets simpler in the X-rays), assume that you have a central ionizing source which looks like:

$$N_\nu \;=\; K_1\,\nu^{-2.3} \quad , \quad \nu > 13.6\,\mathrm{ev}$$

(N_ν below 13.6 eV is not important, of course.)

Assume that you have a pure-H nebula surrounding this source with density $n_\mathrm{H} = 100\,\mathrm{cm}^{-3}$, at temperature $T_\mathrm{gas} = 10^4\,\mathrm{K}$. Assume that the inner 0.1pc around the source is empty.

(a) What must be the value of K_1 for the central source above to produce the same total number of ionizing photons as a star (i.e. a blackbody source) at temperature $T_* = 40,000\,\mathrm{K}$ and $R_* = 7 \times 10^{11}\,\mathrm{cm} = 10\,R_\odot$? Assume this value of K_1 for all subsequent parts of this problem.

(b) Is this spectrum decreasing *faster* or *slower* with frequency, as compared to a blackbody?

(c) Refer to your class notes for the ionization structure (i.e. $n_{\mathrm{H}^+}/n_{\mathrm{H}^0}$ as a function of distance from the star) of an HII region around a star such as that described in part (a).

Before doing any subsequent parts of this problem, *predict* what the ionization structure of this nebula will be, qualitatively, by drawing (by hand) a plot of $n_{\mathrm{H}^+}/n_{\mathrm{H}^0}$ vs. r (where r is the distance from the ionizing source) on top of a plot of $n_{\mathrm{H}^+}/n_{\mathrm{H}^0}$ vs. r for the star from part (a). Describe if you are attempted to show any particular features or differences, and explain why you think they should be there. Do *not* come back and change what you've done here after you've moved on from this part of this problem.

(d) Use the code you produced for your numerical problem to plot the ionization structure of this nebula. Describe in words what the main difference(s) from the HII region around an $O6$ star is/are, and explain why this spectrum gives rise to those differences.

(e) Plot N_n vs. ν at $r = 0.1\,\mathrm{pc}$ (i.e. before it hits the gas), and shortly before the transition region (e.g. at $r = 2.0\,\mathrm{pc}$).

(f) Predict how the He^+ ionization structure will compare to the H ionization structure you plotted in part (c).

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2. Consider a non-radiative shock moving through *molecular* Hydrogen gas ($\gamma = 7/5$) of density $n_{\mathrm{H}_2} = 10^4\,\mathrm{cm}^{-3}$ and temperature $T = 100\,\mathrm{K}$. (This density and temperature applies to the *unshocked* gas).

(a) What is the isentropic sound speed in this gas, in km/s?

(b) What must the temperature of the shocked gas be to appreciably excite the first vibrational state of molecular Hydrogen?

(c) What shock velocity (in km s^{-1}) is needed to produce the H_2 temperature in (b)? (Use a numerical solver for this part if you must.)

(d) What must the temperature of the shocked gas be to do much collisional dissociation of molecular Hydrogen?

(e) What shock velocity (in km s^{-1}) is needed to produce the H_2 temperature in (d)? (Use a numerical solver for this part if you must.)

3. In an HII region near the surface of a molecular cloud, the radiation from an ionizing star has broken through one edge of the cloud, leading to very low densities far to the right in the picture below:

The bulk of the neutral cloud lies to the left of the picture and has number density n_0 (in cm^{-3}). At the ionization front (i.e. the edge of the ionized region, indicated by the solid line), high-energy photons from the star ionize neutral hydrogen, and the ionized gas moves away from the front at velocity u. Note that u is the velocity of the ionized gas in the frame of the molecular cloud and of the star; the velocity of the front itself is not given. Assume that the flow of the gas does not vary significantly with time on the relevant time scales.

(a) With reference to the Euler equations, does the bulk velocity of the *increase* or *decrease* as it flows to the lower density regions to the right?

(b) Does the pressure of the gas increase or decrease to the right? (Ignore the gravity of the star, the self-gravity of the gas, and the gravity of the molecular cloud to the left.)

(c) Knowing that the temperature of the HII region, heated by ionizing radiation from the star, is about $T = 10^4$ K, show that the gas comes off of the ionization front with velocity u of order the sound speed in the HII region.

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