A311 Final, Solo Component

Due Friday, December 16

Do not even look at this until you have turned in the group portion of the final!!

You are allowed to use Spitzer, Osterbrock, a table of physical constants (such as the Particle Data Booklet, or a scientific calculator with constants and the ability to convert units), the knowledge that 1 eV = 1.60×10^{-12} ergs, your class notes, and any problem sets and solutions I have posted on the course web page.Do not use any other references during this final.

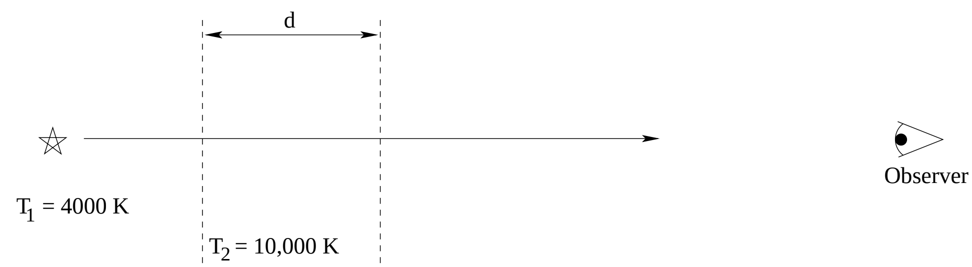
Do not discuss this portion of the final with anybody other than Rob or Rachel until after the due date.

1. Fun with Optical Depths.

(a) Consider a gas cloud of number density $n_{\rm H} = 1\,{\rm cm}^{-3}$. At a given wavelength, the optical depth through the whole gas cloud is τ_ν(initial). If you compress the gas cloud so that its final density is twice its initial density, how does the total optical depth through the cloud τ_ν(final) compare to τ_ν(initial)? (Express your answer as a ratio τ_ν(final)/τ_ν(final).)

(b) Consider the standard interstellar extinction law (shown in class on or around September 23). Is the optical depth between us and the center of the galaxy, taking into account all of the dust twixt here and there, *greater* or *lower* at $\lambda = 2\mu$m than it is at $\lambda = 6000$Å?

(c) Consider a star with a uniform slab of gas between it and the observer. The star has a blackbody spectrum at temperature $T_1 = 4000$ K, and the slab radiates with a blackbody spectrum at temperature $T_2 = 10,000$ K. The observer detects light at 6000 Å.



At some thickness, the specific intensity I_ν of the light detected from the slab will equal the specific intensity of light from the star behind it. At this thickness, what is the optical depth of the slab τ_ν to light at $\nu = 6000$ Å? (Hint: $\tau_\nu \ll 1$).

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2. There are two ways dust can redden starlight. First, it absorbs/scatters more efficiently in bluer light than it does in redder light, so a greater fraction of redder light from the star shines through the dust. Second, if the dust itself emits, it is usually at a lower temperature than a typical star, and hence has a blackbody peak further to the red; this emission then adds fractionally more light to redder wavelengths than it does to bluer wavelengths.

Sometimes, you can disentangle this using *color-color* diagrams. Consider a specific situation: dust at 400 K. Consider the three standard near-infrared broadband filters J, H, and K, which are roughly at wavelengths $\lambda_J = 1.2\mu$m, $\lambda_H = 1.7\mu$m, and $\lambda_K = 2.2\mu$m.

For reference, here are the shapes of the blackbody spectrum;

$$B_\nu = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

$$B_\lambda = \frac{2hc^2/\lambda^5}{\exp(hc/\lambda KT) - 1}$$

(a) What is the peak wavelength $\lambda_{\max}$ where B_λ reaches its maximum for the dust's blackbody spectrum?

(b) What is the *ratio of flux ratios*:

$$\frac{\left(\frac{F_\lambda(J)}{F_\lambda(H)}\right)}{\left(\frac{F_\lambda(H)}{F_\lambda(K)}\right)}$$

for this blackbody? (Evaluate this ratio numerically.)

(c) What is J-H and H-K for this blackbody, where $X \equiv -2.5\log(f_X/f_{0X})$, and where a blackbody at $9,600$ K (i.e. Vega) is *defined* to have J-H=0 and H-K=0? (This definition means you do *not* need any of the values of f_{0X}.)

(d) Consider the standard interstellar extinction curve. If you have 1 magnitude of J-band extinction, how many magnitudes of H-band and K-band extinction do you have, respectively?

(e) What is the ratio $E(J\text{-}H)$ / $E(H\text{-}K)$ you get as a result of interstellar extinction? (The quantity $E(X\text{-}Y)$ is defined as the X-Y you observe through the dust minus the X-Y you would have observed without dust; E stands for "excess".)

(f) Sketch a plot with the vertical axis as H-K and the horizontal axis as J-H. Pick a point on the plot, and label that point "star". This represents the colors of your star (whatever they are). Draw two vectors on this plot originating at the star position, which indicate (1) the direction that the observed color will move as you add more and more dust *extinction*, and (2) the direction that the observed color will move as you add more and more dust *emission* (at T=400 K).

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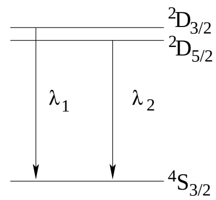
3. Consider the following toy atom (which is nothing real), assuming that *only* the three states indicated exist, and that no significant transitions occur between the $^2D_{3/2}$ and $^2D_{5/2}$ states.



The $^2D_{3/2}$ state has a collisional de-excitation rate of $q_{21} = 3 \times 10^{-8}$ cm^3 s^{-1} (at the relevant temperature), and a radiative de-excitation rate of $A_{21} = 2 \times 10^{-5}$ s^{-1}.

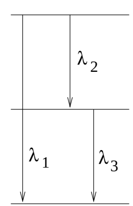
The $^2D_{5/2}$ state has a collisional de-excitation rate of $q_{21} = 2 \times 10^{-8}$ cm^3 s^{-1} (at the relevant temperature), and a radiative de-excitation rate of $A_{21} = 4 \times 10^{-6}$ s^{-1}.

(a) What is the critical density of each transition?

(b) Would you expect the line ratio λ_1/λ_2 to increase or decrease as density increases? (Be careful to associate the right λ with the right state in the picture.)

(c) As drawn, the two ^{2}D states are at a similar energy in comparison to their gap from the energy of the ^{4}S state. If instead we chose two upper states more widely separated in energy, would this make a good density diagnostic? Why or why not?

(d) Consider the collection of states drawn below:



Why would it be preferable to use the ratio λ_2/λ_3 rather than λ_1/λ_3 as a temperature diagnostic? Why?

(e) Will the ratio you chose in (d) increase or decrease with temperature?