A311 Problem Set 1

Due September 9, 2005

1. As we saw in class, the *flux* of a ray crossing an area dA is

$$dF_\nu = I_\nu \cos\theta \; d\Omega$$

since the effective area is reduced by the angle between the area's normal and the direction of the ray.

- **(a)** What is the momentum flux across dA in the direction normal to dA for a given light ray? Recall that the momentum of a photon is $h\nu/c$ and that the energy of a photon is $h\nu$.
- **(b)** Write down an integral expression for p_ν the total flux normal to dA for light of a given frequency, in terms of I_ν and θ.
- **(c)** Consider an enclosure containing an isotropic radiation field. The walls of the enclosure are perfectly reflecting. Write down an integral exprssion for the radiation pressure on the walls of the enclosure. What is the ratio $w = p/u$ between the radiation pressure and the energy density? w is sometimes called the "equation of state" parameter. What is the dimensionality of w?

2. Consider a gas cloud of constant density.

- **(a)** What is the mean optical depth travelled by a photon of frequency ν through this gas? (Note: if the gas is optically thick, a typical photon won't make it all the way through the cloud!) Assume a constant absorption coefficient k_ν.
- **(b)** The *mean free path* of a photon, l_ν, is defined as the average distance a photon of frequency ν can travel before being absorbed. What is the value of l_ν in terms of the parameters of the gas (i.e. j_ν and k_ν)?

4. [**Solo Problem**]

- **(a)** Show that the condition that an optically thin cloud of material can be ejected by radiation pressure from a nearby luminous object is that the mass to luminosity ratio (M/L) for the object be less than $\kappa/(4\pi Gc)$, where G is the gravitational constant, c is the speed of light, and κ is the mass absorption coefficient of the could material (assumed independent of frequency). Recall that Luminosity L is the total rate of energy (in erg/s or J/s) emitted by an object.
- **(b)** Calculate the terminal velocity v attained by such a cloud under radiation and gravitational forces alone, if it starts from rest a distance R from the object. Show that

$$v^2 = \frac{2GM}{R}\left(\frac{\kappa L}{4\pi GMc} - 1\right).$$

- **(c)** A minimum value for κ may be estimated for pure hydrogen as that due to Thomson scattering off free electrons, when the hydrogen is completely ionized. The Thomson cross section is $\sigma_T = 6.65 \times 10^{-25}\text{cm}^2$. The mass scattering coefficient is therefore $> \sigma_T/m_H$, where m_H is the mass of a hydrogen atom. Show that the aximum luminosity that a central mass M can have and still not spontaneously eject hydrogen by radiation pressure is:

$$L_{\text{EDD}} = 4\pi GMcm_H/\sigma_T = 1.25 \times 10^{38}\text{erg s}^{-1}(M/M_\odot),$$

where $M_\odot = 2 \times 10^{33}$g, the mass of the sun. This luminosity is called the *Eddington limit.*

... continued on reverse...

5. A certain gas emits thermally at the rate P(ν) (power per unit volume and frequency range). A spherical cloud of this gas has radius R, temprature T, and is a distance d from earth ($d >> R$).

(a) Assume that the cloud is optically *thin*. What is the brightness (i.e. specific intensity) of the cloud as measured on earth? Give your answer as a function of the distance b away from the cloud center, assuming the cloud may be viewed along parallel rays as shown below.

(b) What is the *effective temperature* of the cloud?

(c) What is the flux F_nu measured at earth coming from the entire cloud?

(d) How do the measured *brightness temperatures* compare with the cloud's tempearture?

(e) Qualitatively, what would be the approximate appearance of the optically think cloud if if were a spherical shell of material (of thickness $\sim 0.1R$) instead of a uniform sphere?

(f) Answer parts (a)-(d) for an optically *thick* cloud.