A311, 2005-10-07

1. **(a)** Without opening Osterbrock, sketch the line ratio:

$$\frac{j_{5007+4959}}{j_{4363}}$$

as a function of temperature. You will find useful the following numbers:

$$
\begin{aligned}
h &= 6.626 \times 10^{-27}\ \mathrm{erg\,s} \\
\hbar &= 1.055 \times 10^{-27}\ \mathrm{erg\,s} \\
k &= 1.381 \times 10^{-16}\ \mathrm{erg\,K^{-1}} \\
m_e &= 9.109 \times 10^{-28}\ \mathrm{g} \\
1\,\mathrm{amu} &= 1.66 \times 10^{-24}\,\mathrm{g} \\
\frac{hc}{4363\text{Å}} &= 4.56 \times 10^{-12}\ \mathrm{erg} \\
\frac{hc}{5000\text{Å}} &= 4.0 \times 10^{-12}\ \mathrm{erg} \\
\Omega(^3P,^1D) &= 2.17 \text{ for [OIII]} \\
\Omega(^3P,^1S) &= 0.28 \text{ for [OIII]} \\
\Omega(^1D,^1S) &= 0.62 \text{ for [OIII]}
\end{aligned}
$$

You will also find this table of einstein-A coefficients useful:

Transition	λ (Å)	$\mathbf{A_{21}}$ $\mathbf{s^{-1}}$
$^1D_2 -^1 S_0$	4363	1.8
$^3P_2 -^1 S_0$	2331	7.8×10^{-4}
$^3P_1 -^1 S_0$	2321	2.2×10^{-1}
$^3P_2 -^1 D_2$	5007	2.0×10^{-2}
$^3P_1 -^1 D_2$	4959	6.7×10^{-3}
$^3P_0 -^1 D_2$	4931	2.7×10^{-6}

(b) Suppose your uncertainties on your flux measurements for *all* lines is 1% of the flux of the 5007 line. Over what range of temperatures is the OIII line ratio a useful temperature diagnostic?

(c) The 4363Å line tends to be a lot fainter than the 5007Å and 4959Å lines. Why can't we use the easier-to-measure 4959Å/5007Å line ratio as a temperature diagnostic?

2. **(a)** What are the critical densities for the 4363, 5007, and 4959 lines? This is like redoing problem 3 from the last homework, only now you have to calculate the cross-sections, and you have to do it right. (I did it wrong.) All the data you need is in the previous problem.

(b) As density increases, if you do not take this into account will you tend to *overestimate* or *underestimate* the temperature of a nebula by using the [OIII] line diagnostic?

3. You observe a nebula which you believe to be optically thin. Assume that it has Solar abundances ($n_{\mathrm{He}} = 0.1 n_{\mathrm{H}}$, $n_{\mathrm{O}} = 7 \times 10^{-4} n_{\mathrm{H}}$). Assume that you've figured out that it is at $T = 10^4\,\mathrm{K}$ and $n_e = 10^2\,\mathrm{cm^{-3}}$. Finally, assume (perhaps rashly) that all of the oxygen in this nebula is doubly ionized.

You measure an [OIII] 5007Å line flux of $4 \times 10^{-6}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$ coming from the nebula. If this nebula is 1 kpc ($= 3.086 \times 10^{21}\,\mathrm{cm}$) away, what is the mass of the nebula?

. . . continued on reverse. . .

4. Show that you would classically expect the collisional cross-section between an electron and an ion to go as v_e^{-2}. Do this in a very back-of-the-envelope manner by estimating:

- how the maximum force between the object depends on the impact parameter (closest pass between the electron and ion if the electron's path weren't diverted);
- how the "interaction time" between the ion and electron for the force to be anywhere near your estimate depends on v_e;
- how the change in electron momentum corresponds to the previous two things;
- how a change in electron momentum corresponds to a change in electron kinetic energy;
- that a "major collision" happens when ΔE/E for the electron is substantial (assume of order 1, although the actual value for this ratio doesn't matter for purposes of determining the v_e dependence).

This should let you estimate what impact parameter b is needed for a given v_e to effect an energy chagne. The cross section will then be something like $\pi\, b^2$.