# A311 Problem Set 4

Due Monday, November 7, 2005

**1.** Calculate the expected ([OIII] $\lambda = 5007 + \lambda = 4959$)/H$\beta$ flux ratio observed from an HII region. Assume that the density is $n_{\rm H} = 10^2\,{\rm cm}^{-3}$; that abundances are solar ($n_{\rm He} = 0.1 n_{\rm H}$ and $n_{\rm O} = 7 \times 10^{-4} n_{\rm H}$); that the gas temperature is $T = 10^4\,{\rm K}$; and that the star is hot enough that the $He^+$ region is nearly the same size as the $H^+$ region (e.g. an O6 star; Osterbrock Figure 2.4).

Hint: H$\beta$ is a recombination line of Hydrogen, and the [OIII] lines are collisionally exited lines we talked about a lot during our week of temperature diagnostics. Assume that the recombination cascade of Hydrogen goes through the 4-2 transition that yields H$\beta$ 12% of the time (which should be a good Case B value for 10,000K).

- **(a)** Calculate the expected ratio. If you want to know if the ratio you calculate is sane, look at various tables and plots throughout Osterbrock that show model calculations and observed values.
- **(b)** How sensitive is your ratio to $n_{\rm H}$?
- **(c)** How sensitive is your ratio to the gas temperature $T$? (You will want to consider Osterbrock Chapter 4 and Table 4.2 to see if/how that 12% figure varies with $T$).
- **(d)** Qualitatively, how sensitive should your ratio be to $L_*$, the overall luminosity of the ionizing star (keeping everything else constant)?
- **(e)** Qualitatively, how sensitive should your ratio be to $T_*$, the temperature of the ionizing star? (Think about lowering the temperature of the star.)

**Note:** If the $\alpha_B$ on the last line of Table 2.1 for a gas temperature of 20,000 K is $2.52 \times 10^{-13}\,{\rm cm}^3\,{\rm s}^{-1}$, then I think that is an error. I *believe* that the right value is $1.43 \times 10^{-13}\,{\rm cm}^3\,{\rm s}^{-1}$, but I need to check this.

**2.** **[Solo Problem]**

- **(a)** What is the total mass of Hydrogen $M_{H+}$ that an O6 star keeps ionized in its Strömgren sphere? Assuming a pure-hydrogen nebula of total density $n_H = 1\ {\rm cm}^{-3}$. How does this mass compare to the mass of the star? (Note that the mass of the sun is $M_\odot = 2 \times 10^{33}$ g.)
- **(b)** Answer the same question for a density $n_H = 10^3\ {\rm cm}^{-3}$.
- **(c)** Sketch a plot of $\log(M_{H+})$ vs. $\log(n_H)$, where $M_{H+}$ is the total mass of Hydrogen kept ionized by an O6 star and $n_H$ is the total density of Hydrogen (including $H$ and $H^+$). (This should be an easy plot to draw, so do it by hand— no computers needed.)

**3.** Consider an HII region in equilibrium about an O6 star. Assume that the gas temperature in the HII region is $10^4\,{\rm K}$. Assume that the neutral Hydrogen gas outside it has a temperature $T \simeq 10^2\,{\rm K}$ and density $n \simeq 10^3\,{\rm cm}^{-3}$.

- **(a)** By considering the pressure balance at the boundary between the HII and HI regions when the whole system has reached equilibrium, what will the density $n$ inside the HII region be?
- **(b)** What will the size $r'_S$ of the Strömgren sphere be at this point?
- **(c)** Given that an HII region expands at a rate of roughly 10 km ${\rm s}^{-1}$, approximately how long will the HII region be expanding? How does this compare to any other time scales given the physical situation, and what can you conclude about HII regions from this? (Hint: there are two obvious relevant physical time scales, and neither is the age of the universe.)