A311 Problem Set 5

Due Friday, November 18, 2005

1. (Solo Problem) Consider two regions of ionized gas. Both have exactly the same gas density, and are around identical stars (same luminosity, same temperature). Cloud A has a *higher* heavy element abundance than Cloud B. (In both cases, they're small compared to H and He, of course.) Qualitatively, how do the gas temperatures of the two clouds compare? Explain.

2. (Solo problem.) Consider the following toy model for the interstellar medium (ISM) of our galaxy as a whole. Assume that the ISM consists of 10^{10} $\mathrm{M}_\odot$ of neutral Hydrogen (ionization fraction 10^{-4}), where $\mathrm{M}_\odot = 2 \times 10^{33}$g is one solar mass. Assume that this gas everywhere has density $n = 1$ cm^{-3}.

If the ISM cools according to the standard HI region cooling curve (yes, which depends on elements other than Hydrogen, but only small amounts of them), is heated entirely by from the kinetic energy of Type II supernovae, and is at a uniform temperature everywhere, what would the temperature of the interstellar medium be? Assume that a Type II supernova releases 10^{51} erg of kinetic energy, and that the galaxy hosts one Type II supernova every 100 years.

(Obviously, this is a too simple model of the ISM; there are molecular clouds of much higher density and lower temperature, as well as both the "cool" and "warm" phases of the neutral atomic medium, and ionized regions.)

3. The viscous stress tensor can frequently be written as:

$$\pi_{ik} = \mu \, D_{ik}$$

where μ is the coefficient of sheer viscosity, and D_{ik} is the deformation rate tensor:

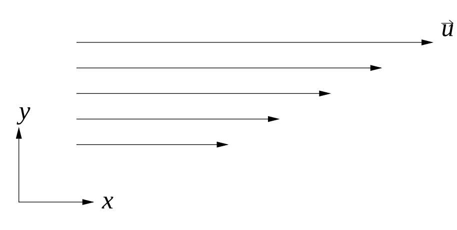
$$D_{ik} = \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3}(\vec{\nabla} \cdot \vec{u})\delta_{ik}$$

Navier-Stokes momentum equation is:

$$\rho \frac{D\vec{u}}{Dt} = -\vec{\nabla} P - \rho \vec{\nabla} \Phi + \mu \vec{\nabla} \cdot \overleftrightarrow{D}$$

This is a "$ma = F$" equation. The term $\mu \vec{\nabla} \cdot \overleftrightarrow{D}$ is the divergence in the viscous force. You can use the divergence theorem in reverse to see that the viscous force is the the viscous force μD_{ik} on each "wall" i, summed over all of the "walls" of a fluid element.

Consider a plane parallel flow with $\vec{u}$ entirely in the $+x$-direction, and with translational symmetry along the z-axis, but with variation along the y axis (e.g. because there may be a border (river bottom?) far in the $-y$ direction). Approximate the change in u_x with y as linear, as drawn.



(a) In this problem, which components of D_{ik} are nonzero, and what are their values in terms of derivatives of $\vec{u}$?

(b) What is the viscous force (magnitude and direction) in terms of the nonzero components of D_{ik} (written out as partial derivatives of $\vec{u}$) on a fluid element?

...continued on reverse...

(c) Consider a fluid element, with (among others) two boundaries parallel to the $x - z$ plane: a "top" boundary at higher y, and a "bottom" boundary at lower y. Particles of u_x higher than that of our fluid element will cross the top boundary, and particles of lower u_x will cross the bottom boundary. What is the rate at which particles cross each of these boundaries? Express your answer in terms of T, the temperature of the fluid, m, the mass of each atom, n, the number density of the fluid, ΔA, the area of the fluid element boundary, and fundamental constants.

(d) Each particle that enters our fluid element of interest will travel an average distance before interacting with particles that "belong" to our fluid element, and therefore transferring its momentum to our fluid element. What is this typical mean distance, in terms of m, T, n, and σ (the particle interaction cross section)?

(e) Using that mean distance, what approximately is the amount of $x-$momentum that each particle will transfer? Express your answer in terms of the usual quantities plus derivatives of $\vec{u}$.

(f) Continue the consideration of this momentum transfer to estimate a value for μ in terms of m, T, n, σ, and fundamental constants. Remember that force is just time rate of change of momentum!

(g) Is there any quantity missing from the estimate of μ whose absence *might* be surprising, intuitively thinking about what would cause viscosity? Why, in words, does it make sense that this quantity is missing?