A311 Numerical Problem Set

Due Wednesday, December 7, The Last Day of Class, Absolutely

Note: I will be out of town from Thanksgiving Break (Nov 20) through December 5. I will be in e-mail contact most of that time.

Sort-of Solo Problem Set *You can talk with each other about how to code, how to program, how to plot from within a program, how to compile. You can talk with each other about the general aspects of the algorithm. However, you each need to write your own code. Use common sense; if you are getting to the point where you're telling somebody else how to write the program, you've gone too far.*

1. Write a program in the language of your choice to calculate the ionization structure of a pure Hydrogen HII region around a hot star. You should divide the nebula into a series of thin spherical shells. Solve the ionization equilibrium equation to determine the ionization fraction n_{H^+}/n_H $(= 1 - n_{\mathrm{H}^0}/n_{\mathrm{H}})$ in each shell, and then determine how many photons as a function of frequency are absorbed in that shell. Work from the inside out, passing the number of photons at each frequency on to each next shell. The shell width should be narrow "enough" (in particular, make sure that there are a number of shells across the transition region from nearly-fully-ionized to nearly-fully-neutral). Continue out in radius until you are past the transition region.

Input parameters (defined as constants or something you pass to the code, but in any event easily changeable) should include:

- n_{H}, the density of the nebula (start at $n_{\mathrm{H}} = 10^2\,\mathrm{cm}^{-3}$)
- T_* and R_*, the temperature and radius of the star (start with values appropriate for an O6 star)
- T, the gas temperature (just assume 10^4 K)
- r_{inner}, an inner radius of the nebula: assume that it is empty vacuum between the star and r_{inner}. You can set r_{inner} equal to the thickness of your shell, for convenience. If you want to be extreme, and have the computing power, make the thickness of your shell equal to r_*.
- Other things you need that I've forgotten at the moment.

What you should produce are:

- A plot of $n_{\mathrm{H}^+}/n_{\mathrm{H}}$ as a function of r, the distance from the center of the nebula.
- A plot of N_ν and/or L_ν at various positions through the nebula. The first should be right at the surface of the star; there should be a few throughout the nebula; and there should be a few across the transition region.

Repeat this for a range of parameters; try a few different densities, and a few different types of central star. Turn in your code (suitably commented and readable) and your plots (suitably labelled and notated, with any calculations or notes you deem appropriate). You can e-mail the code if you wish.

2. Enhance your program so that it includes the ionization of HeI to HeII. Don't worry about doubly-ionized Helium. An additional parameter should be $n_{\mathrm{He}}/n_{\mathrm{H}}$, the Helium fraction of the gas (using 0.1). Your plot of ionization fraction should plot *both* $n_{\mathrm{H}^+}/n_{\mathrm{H}}$ and $n_{\mathrm{He}^+}/n_{\mathrm{He}}$, similar to Osterbrock Figure 2.4.

3. **Extra Credit– not required!** Now allow Helium to be doubly ionized. Look at Osterbrock Section 2.5 for hints on how to deal with this. This makes things enough harder that I haven't yet thought through myself how to do it, but it should be doable.

4. 10^{10} **Points of Extra Credit.** Reimplement CLOUDY.