Energy & Matter: Our Quantum World

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Preface

This text is designed as an introduction to the theory of quantum mechanics for college students who have not had calculus, nor who have had any prior college-level courses in physics. The majority of quantum texts you can find out there either assume that the student is already familiar with calculus (and perhaps linear algebra), or are popular-level treatments of the topic that have nearly no rigorous mathematical content at all. This text tries to straddle the difference. The target audience are students who might take an algebra-based introductory physics course. No calculus is assumed, nor is any linear algebra. (The text will eventually use a very small amount of linear algebra, but it will introduce that, so the reader need not know it coming in.) However, it does give a rigorous introduction to quantum mechanics, and does not shy away from showing the mathematics of the theory where that is accessible to students with this sort of mathematical background.

Because quantum theory represents a way of looking at the world that is completely at odds with our intuition— so much so that physicists still debate how properly to interperet such things as the "measurement problem"— anybody who has had no prior exposure to quantum physics will find this material conceptually challenging. That is as it should be! The goal of a university course, particularly an introductory university course, should be to expand your mind, to make you work out parts of your brain that you might not even have known that you had. However, this does mean that if a student comes into the course uncomfortable with the concepts behind algebra at the advanced high-school level, they may be overwhelmed. If solving two systems of equations is something that you're shaky on, and if you're not comfortable with the *meaning* of an algebreic variable as a stand-in for something that we may or may not know, then you will find yourself at a disadvantage as you struggle with those concepts while also facing the new concepts of quantum physics. I want to emphasize that this text is *not* designed for physics students or math students in particular; it's designed for all liberal arts university students. I simply expect that those students will take seriously the on-paper prerequisite found at most universisties of having matered high school algebra through the grade 11 level.

The course does require students to try to deal with mathematics at an abstract level. Students tend to be much more comfortable with math when it is concrete.

I have found in all introductory physics courses that when faced with a problem requiring algebraic manipulations, students like to plug values into variables as soon as they can, and then manipulate the numbers. Those of us with more experience recognize this as a trap, for the resulting process is much more error prone and hard to follow than if one had solved the equation symbolically first. Students, however, seem to prefer to remove any abstractions as soon as possible. It's worth trying to train students to work with the algebra at an abstract level, only plugging in numbers when they absolutely can't avoid it any more (e.g. to determine a numerical result). At a higher level, this text *does* introduction the notion of operators, but doesn't always describe exactly the mechanics of those operators. Students will all be familiar with the square root; they know how it behaves on a number. The square root, of course, is an operator. If students aren't intimidated by it, it's simply because they're familiar with it, and because there's a button on their calculator that will perform the operation on a number. To explore quantum physics, this text will introduce operators as "something that does something to something", perhaps leaving the second "something" completely abstract. One need not know the detailed numerical representation of an operator in order to know that a given operator will extract the eigenvalue from one of its eigenstates. Students will often find this a difficult concept to grasp, but because it is so powerful (allowing you to solve and do things often without having to learn the details), the text does not shy away from it.

While many "modern physics" courses designed for second-year physics majors start with the Planck spectrum, and move quickly to the one-dimensional Schrödinger Equation, this text starts, after a quick background in basic physics (as no prior university level physics is assumed), with the spin-1/2 system. Of course, dealing with the Planck spectrum and with the Schrödinger equation requires calculus, so that wouldn't be an appropriate place to start for an algebra-based course. However, I believe that starting with the spin-1/2 system may well be a better way to introduce students to the concepts behind quantum mechanics. It's a very simple system, as there are only two states available. The concepts, while counter-intuitive, may be explained and understood, and the mathematics behind them may be explored with only algebra in your background. Concepts such as orthogonality and eigenstates are easily obscured amongst the slog through integrals that happens when solving the differential Scrhödinger equation as your first introduction to quantum physics. With the spin-1/2 system, however, there is much less to distract you.

This text does eventually inroduce the Scrhödinger Equation, but because students can't be assumed to know any differential equations it leaves the kinetic energy operator entirely abstract. As such, students won't be solving the Schrödinger Equation. However, they will explore some of the consequences of some of the famous known solutions, including the square well and the simple harmonic oscillator. The text then introduces the Hydrogen Atom. After having focused for so long on spin angular momentum (in the spin-1/2 system), it describes how orbital angular momentum in the Hydrogen Atom solutions to the Schrödinger Equation naturally gives rise to the sturcture of the Periodic Table of the Elements.

This text was written for the foundation course *Energy and Matter* at Quest University Canada. That course has several different versions; "Our Quantum World" is one of them.

A note about commas

You may be offended to find me placing commas and periods outside of quatation marks. I realize that I'm naughty, but as a computer programmer I can't help but notice that standard usage is *wrong*, and leads to a pandemic misquoting. Consider the following sentence: many people have read "A Tale of Two Cities", a novel by Charles Dickens. Standard usage would have me put the comma inside the quotation marks, but the comma is *not part* of the title. If the purpose of the quotation marks here is to set off the title from the rest of the text, then you're misrepresenting the title by including things inside the quotation marks that aren't part of that title. When I'm forced to, I use the standards. However, I would dearly love to see the standards change to something more logical and reasonable.

End of rant.

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