Chapter 2

Energy

"Energy" is an extremely loaded term. It is used in everyday parlance to mean a number of different things, many of which bear at most a passing resemblance to the term as used in physical science.

At its core, energy is a mathematical construct that has turned out to be extremely useful. It shows up always with the same dimensionality, but in different forms. In a physical system, you can identify the forms of energy that are present, and calculate a number that represents the amount of energy there is for each of these forms. Ultimately, though, energy is just a mathematical construction that we calculate. What makes it so useful, however, is the observation that in all successful theories of physics thus far, *energy is conserved*. We could just as easily name and calculate an unending variety of other quantities for physical systems, but few are quite so useful as energy. If you take into account all of the various forms of energy in a complete system, you neither create nor destroy it. That is, in any interaction, the total amount of energy afterwards is exactly the same as the total amount of energy beforehand. Any energy lost by any part of the system must have been gained by another part of the system, and vice versa.

2.1 The Units of Energy

The SI unit for Energy is the joule, usually abbreviated J. One joule is equal to one kilogram meter squared per second squared:

$$1\,\mathrm{J} \;=\; 1\,\frac{\mathrm{kg\,m^2}}{\mathrm{s^2}}$$

Anything that is energy can be written as a number of joules. However, this isn't the only unit for energy. You are probably more familiar with another unit, the kilocalorie. (A kilocalorie is what is reported as mere *Calories* in food. The name is unfortunate, for there are 1000 calories in one Calorie; you can easily see how this would lead to confusion.) There are 4,184 joules in a kilocalorie; you can use this with the unit factor method (Section 1.3 to convert between the two forms of energy.)

A unit for energy that will be used more often in this course is the *electron Volt*, abbreviated eV. The conversion rate to joules is:

$$1\,\mathrm{eV} \;=\; 1.6022 \times 10^{-19}\,\mathrm{J}$$

The electron volt is a unit of energy well suited to the processes that happen in atoms. For example, it takes 13.6 electron volts to rip the electron off of a Hydrogen atom. This is a far more convenient number to use than 2.18×10^{-18}, the corresponding number of joules. It is important to remember, however, that the electron volt *is* the same dimensionality as joules, and that you can freely convert back and forth between the two. The eV is *not* a unit of voltage, nor, despite its name, is it specific to the electron; you can measure the energy of *anything* in eV. For example, if you consume 2,000 kilocalories worth of nutrition each day, it would be true, if not terribly illuminating, to say that you consume 5×10^{25} eV worth of food energy every day.

2.2 Forms of Energy

There are a number of forms of energy, but *most* of them can be reduced to either *kinetic energy* (energy of motion) or *potential energy* (energy of relative position). These two are discussed in greater detail below.

Heat energy, more accurately called *thermal energy*, is a form of energy that a bulk substance can have. As the temperature of an object goes up, its thermal energy content also goes up. Ultimately, however, thermal energy is just a form of kinetic energy. It is the vibrations of the molecules that make up the substance, or, in the case of the gas, the motions of the molecules zipping about that make up this thermal energy. When you heat water up, it gets hotter because the average speed at which water molecules are vibrating goes up. Indeed, that is what it *means* to say that water is hotter.

Internal energy is a catch-all term sometimes used to indicate energy that you're not keeping track of. As it sounds, this is energy that is, somehow, stored inside an object. In reality, this energy is made up of kinetic and potential energy. It may be that things inside the object are moving around, and thus the internal energy you're talking about is in the form of kinetic energy. You can, if you insist on painting with a broad brush, treat thermal energy as a form of internal energy. As another example, it may be that inside your object there are springs or other things that, as they move around, acquire potential energy as a result of their relative positions.

Chemical energy, sometimes called *chemical potential energy*, is, as the latter name suggests, just a special form of potential energy. It represents the energy that you could get out of a substance by performing chemical reactions with it. The chemical energy stored in gasoline may be treated as a form of internal energy, which you can extract and turn into other forms by burning that gas. On the microscopic level, what you're doing is rearranging the atoms into different molecules. That is, you're putting all of the atoms into different positions relative to each other.[1] Because potential energy is the energy of relative position, this means that you're changing the potential energy of all of these atoms.

Mass is, itself, a form of energy, leading to the term *mass energy*. Using Einstein's famous equation $E = mc^2$, you can convert from mass to other forms of energy. In chemical reactions, the amount of mass that is converted to or from energy is tiny— roughly one part in a billion. This is tiny enough that chemists will talk about the "conservation of mass", even though this is not strictly true. In nuclear reactions, however, the amount of mass that is converted to energy can be appreciable, approaching a percent. In matter-antimatter reactions, it is possible to convert *all* of the mass of reactants into other forms of energy.

Light energy, or more generally *radiation*, is energy in particles that are moving so fast (up to as fast as possible, in the case of light!) that their kinetic energy is much higher than their mass energy, if any.

Dark energy is in fact not energy in the classic sense of the word, but is the name given to the mysterious substance that fills the Universe and is driving its expansion to accelerate. We know next to nothing about dark energy, and we certainly don't know how to convert it to other forms of energy.

2.2.1 Kinetic Energy

If an object with mass m is moving with speed v, then the amount of kinetic energy that object has is

$$KE = \frac{1}{2} m v^2$$

. If you look at the dimensionality of this equation, you will see that on the right we have mass times length squared divided by time squared. In SI units, that would be $\mathrm{kg\,m^2\,s^{-2}}$, which is the Joule. It is comforting to see that this equation does give us the right units for energy. This equation only works as long as the speed v is a lot less than the speed of light. Once the speed approaches the speed of light, you have to take into account Relativity, and things become more complicated. Why the $\frac{1}{2}$? The

[1] Later, we'll see that talking about the position of particles in atoms is a bit troublesome, but for now this description is reasonable.

answer may not satisfy you: because that's what works. You can derive this from forces using a little bit of calculus, but even that derivation requires other definitions that may seem arbitrary. Ultimately, we've found that if we use this formula for kinetic energy, rather than something else times $m\,v^2$, the notion of conservation of energy works.

It is also possible to have kinetic energy if you are at rest: you can have *rotational kinetic energy* if you are rotating. However, at the microscopic level, ultimately this is the same thing. Imagine a ball that's at rest, but rotating. If you think about each little piece of the ball— each molecule in the ball, if you will— the ones that are not right on the axis of rotation are in fact themselves moving about the center of the ball. The ones closer to the axis are moving slower than the ones farther away. What we call rotational kinetic energy is just a way of summarizing this motion of all of the little pieces of the ball.[2]

2.2.2 Potential Energy

Potential energy is energy of relative position. Except in esoteric situations where general relativity and quantum field theory tentatively approach each other, the absolute value of potential energy doesn't matter. All that matters are the *differences* in potential energy as particles rearrange themselves into different relative positions. This means that you could add any constant (with energy units) you want to the potential energy of a system, and, as long as you don't change the constant you're using partway through a problem, all of your energy calculations will come out right. Frequently, but not always, we choose the constant such that the potential energy is zero for particles that are infinitely far away from each other. This convention makes sense; you don't want to have to think about having some energy to carry around for a particle that is so far away that it's not meaningfully interacting with any of the particles you do care about.

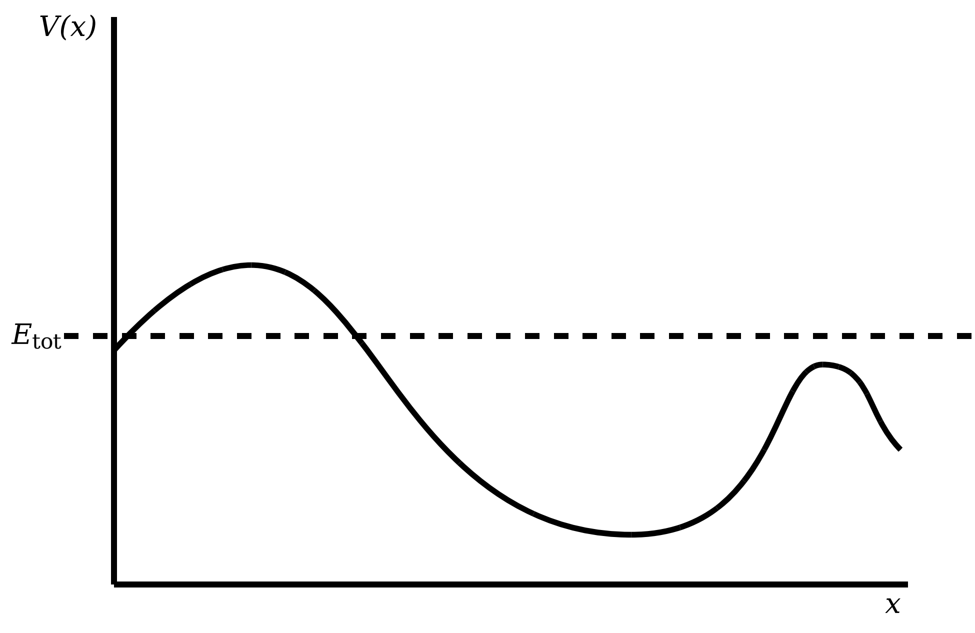
Technically, you can't talk about the potential energy of a single object. Really, the potential energy is in the interaction of that object and another object. To be proper potential energy, it must depend only on their positions relative to each other. It doesn't make sense to talk about "the potential energy of the Earth". However, it does make sense to talk about the potential energy of the Earth-Sun system.

Sometimes, we can make the approximation that one particle is much smaller than everything else it is interacting with. In that case, we will talk about the potential energy *of* that particle. For example, if you lift a ball off of the ground, as that

[2]The formula for rotational kinetic energy is $\frac{1}{2}I\omega^2$. I is the *moment of inertia*; it depends on the mass, size, and geometry of the object. ω is the *angular velocity*, in radians per second. It is equal to 2π times the number of rotations per second the rotating object is making. You will read more about I and ω in Chapter 3.

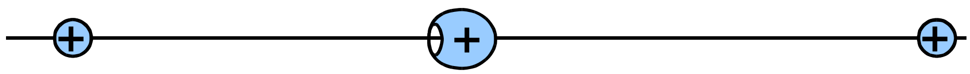
ball and the center of the Earth get farther away from each other there is more and more potential energy in the gravitational interaction between the ball and the Earth. However, the gravity of the ball on the Earth is extremely unimportant to the Earth, whereas the gravity of the Earth on the ball is extremely important to the ball. As such, we can treat the ball as a particle moving within the "fixed potential of the Earth". We then say that the ball has a certain amount of potential energy based on its height above the ground. Implicitly, this is really the potential energy in the interaction of the ball and the Earth, but it is more convenient to treat it as the potential energy of the ball, with the understanding that we're working in the (very valid) approximation that the ball is much smaller than the rest of the system (i.e. the Earth).

Different interactions (i.e. different forces) have different functional forms for potentials. For the moment, you won't need to use any of them. If you have had physics before, you may know some of them. For an arbitrary force or combination of forces, you could construct a potential energy function $V(x)$. It is useful to think of an analogy between a particle moving in a potential and a car rolling about on hilly ground. Suppose that $V(x)$ had the following form:

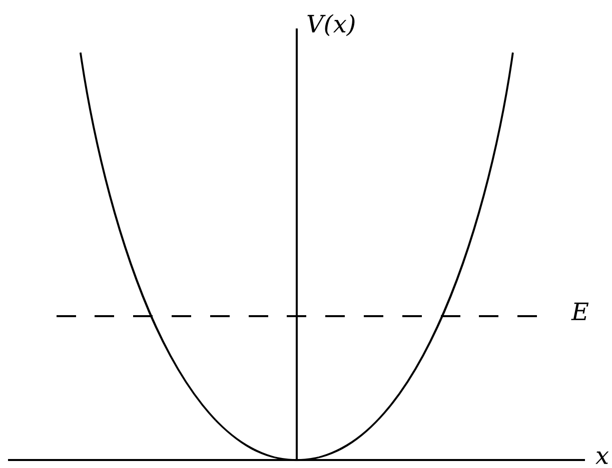


The dashed line on the plot indicates the total energy available to the particle. Imagine that instead of potential energy, the vertical axis where the height of hills, and imagine that the particle is a car. When the car is at a lower point, it has less potential energy, and thus more kinetic energy, and thus is moving faster. The car cannot get to places higher than the dashed line: it's not moving fast enough to make it that far up the hill. By thinking about potential energy in this manner, you can visually get an idea for how particles will move around in a given potential, even if you don't know all of the classical physics needed to work it out.

As another example, suppose you have a wire with two positive electric charges fixed to it. Sliding smoothly along the wire is a bead that also has a positive electric charge on it:



Positive electric charges will repel each other. As such, if the bead will be pushed away from the two positive charges at either end of the wire. Call x the position of the bead along the wire, with $x = 0$ the exact center of the wire. There will be a potential energy function $V(x)$ for the interaction between the bead and the two charges on either end of the wire. To make things more interesting, let's suppose that the bead has some total energy E that is *greater* than the minimum of the potential $V(x)$.



The minimum of the potential energy is where the bead "wants" to be. In this case, the bead is pushed away from the positive charge at either end. If you imagine a ball rolling in this potential, it would experience the same thing; it would want to move towards the center if it were up either side of the potential well. However, looking above at the picture of the bead on the wire, the bead makes no actual motion *down* in space; it's only moving to lower potential. Notice that we've chosen to make $V(x) = 0$ at the center of the wire. Remember that that is completely arbitrary; we could add a constant to the potential energy, and it wouldn't make any difference. (We would have to add the same constant to the total energy of the particle to keep things consistent, however!)

What happens if the bead is at $x = 0$? We can see that it's potential energy $V(x)$ is equal to zero. However, its *total* energy is something greater than that. That means that the bead must have some other form of energy. As we've defined the system, the only other form of energy the bead could have is kinetic energy. This means that if the bead really does have energy E as indicated on the plot, it *must* be sliding either to the left or to the right if it's at $x = 0$. Indeed, at any x, it will satisfy $\frac{1}{2}mv^2 + V(x) = E$.